1. Design a SAT problem with the variables w, x, y, and z that evaluates to true for the truth assignments that have exactly one of w, x, y, or z equal to true.

2. Prove that (e) is not Turing-decidable:

(a) Given M_a, w: Does M_a halt on input w?

(b) Given M_b: Does M_b halt on input ε?

(c) Given M_c: Is there any string on which M_c halts?

(d) Given M_d: Does M_d halt on every string?

(e) Given two TM's M_1 and M_2: Do they halt on the same input strings?

Announcements:

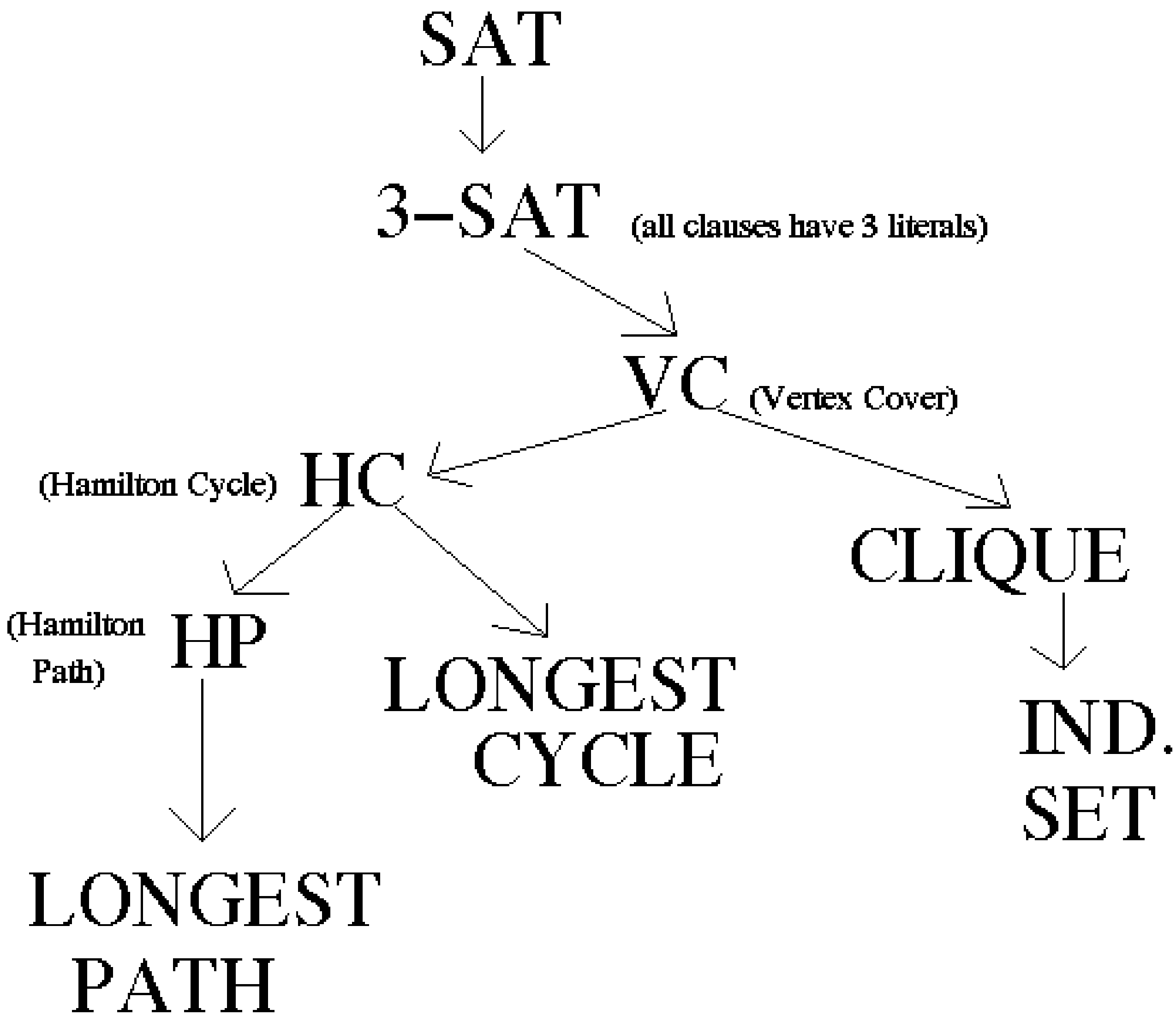
Assignment #5 is due on Friday at the beginning of class.

Please do course evaluations.

Office hours for final exam:

Tuesday August 1: 12:30pm
Thursday August 3: 12:30pm
Friday August 4: 12:30pm

Sunday August 13: 3pm-6pm, Room TBA



History of NP-completeness Reductions

A set $S \subseteq V(G)$ is a vertex cover if every edge of G has at least one vertex in S.

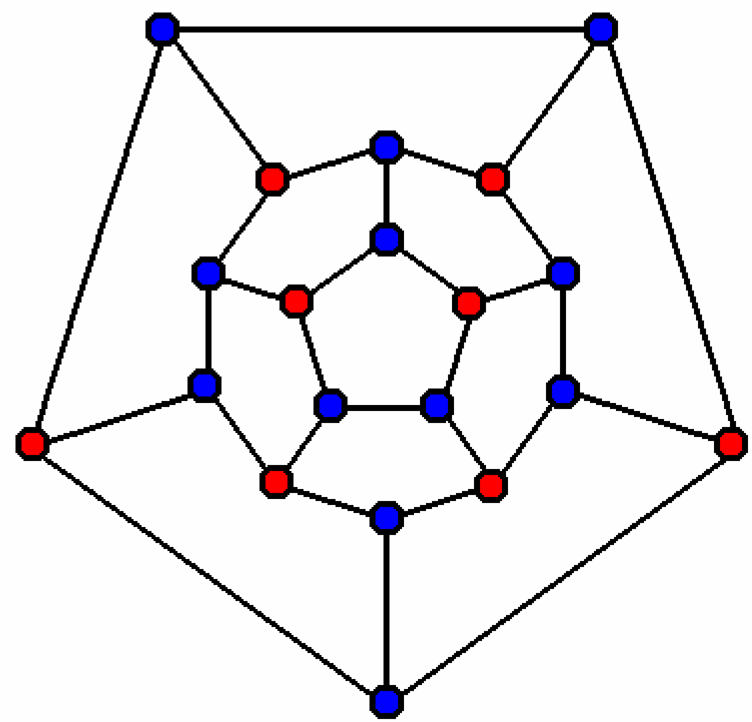
VERTEX COVER:

Given: G, k

Question: Does G have a vertex cover of order k?

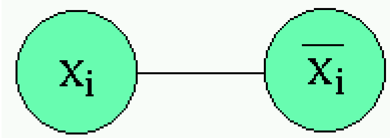
Blue: vertex cover

Red: independent set



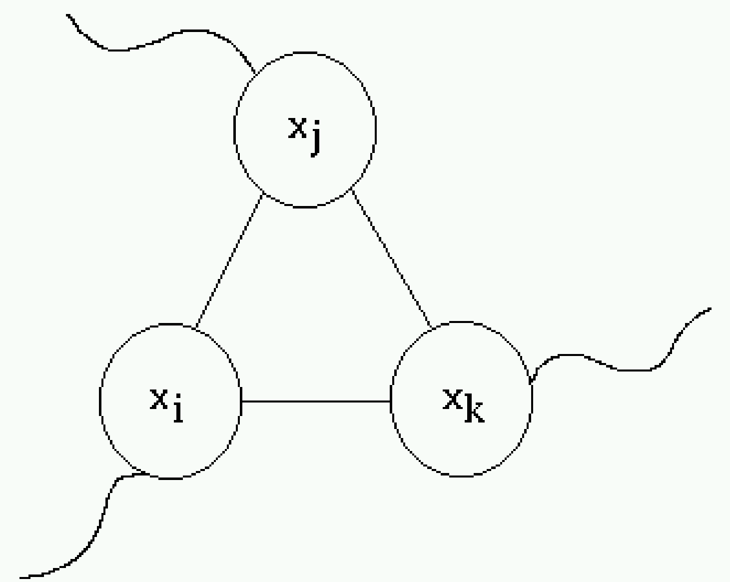
To solve 3-SAT using vertex cover:

1. For each literal x_i, include:



2. For each clause (x_i, x_j, x_k) use a gadget:

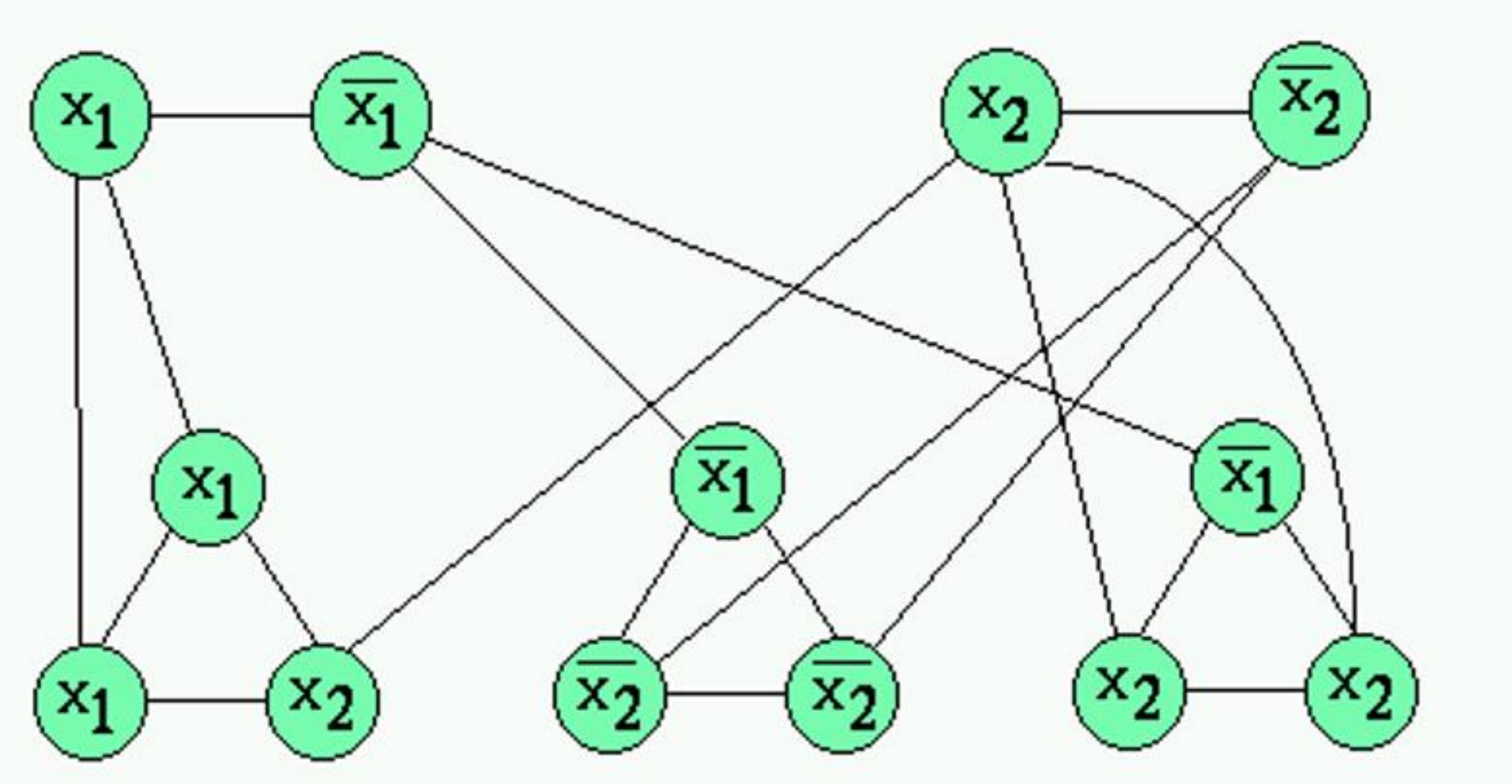
Each white vertex connects to the corresponding green one.



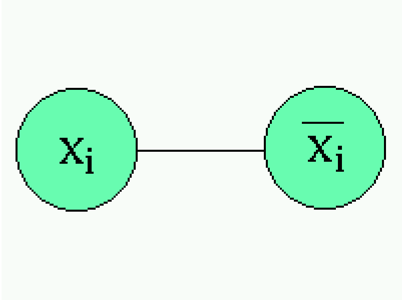
Pictures from:
http://cgm.cs.mcgill.ca/~athens/cs507/Projects/2001/CW/npproof.html

3-SAT Problem:

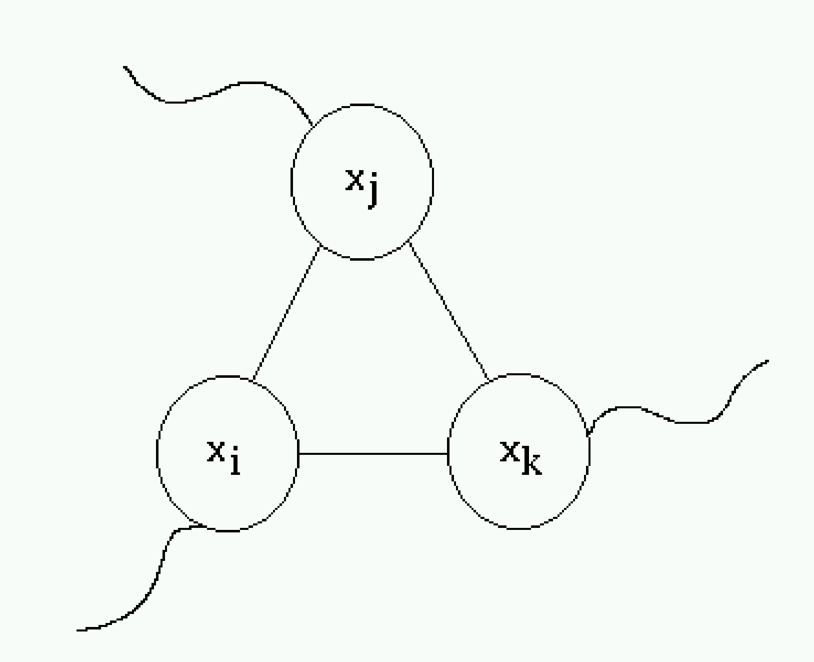
(x1 or x1 or x2) AND (¬x1 or ¬ x2 or ¬ x2) AND (¬x1 or x2 or x2)



At least one vertex from is in the vertex cover.



For each gadget, at least 2 vertices are in the vertex cover:

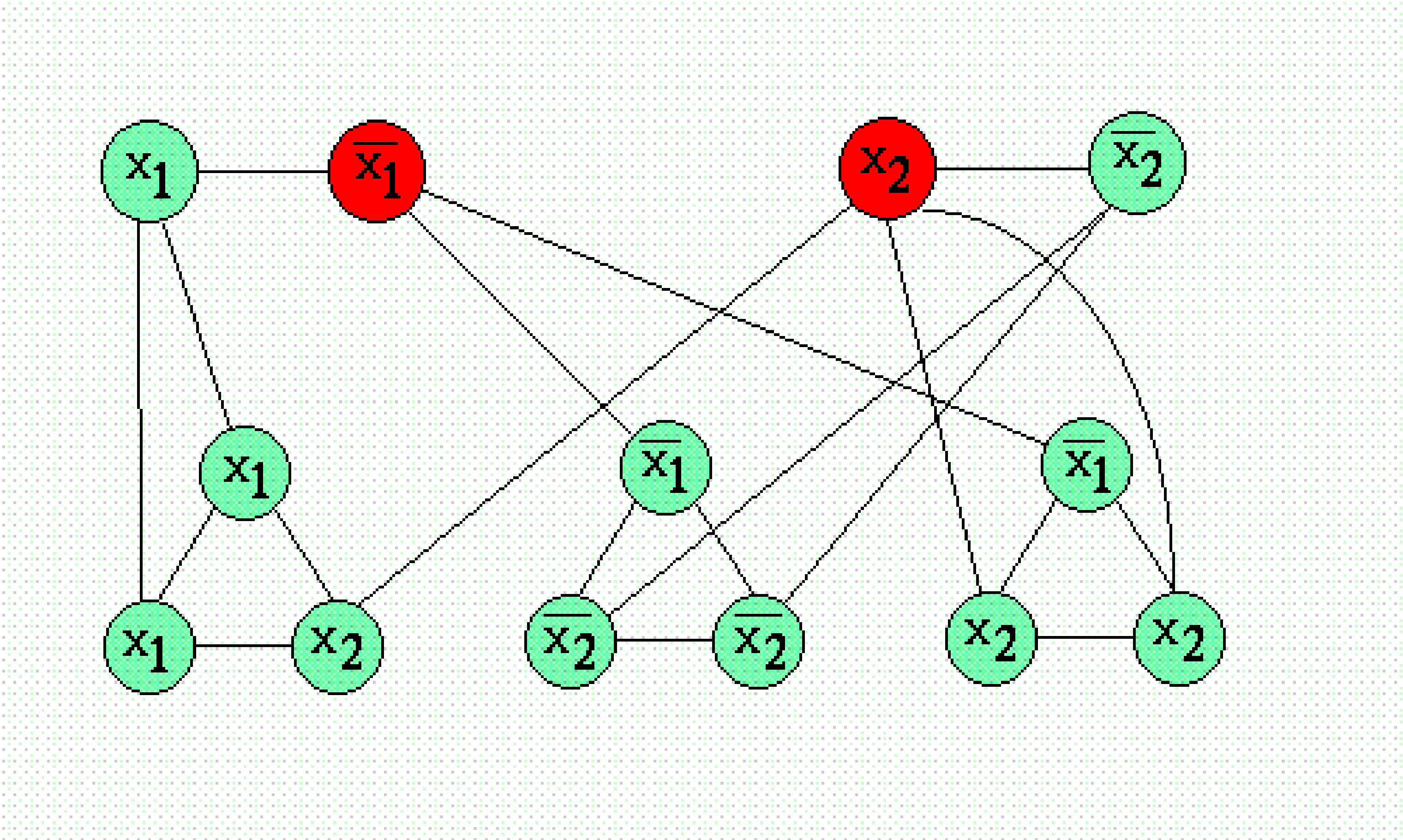


Number of variables: n

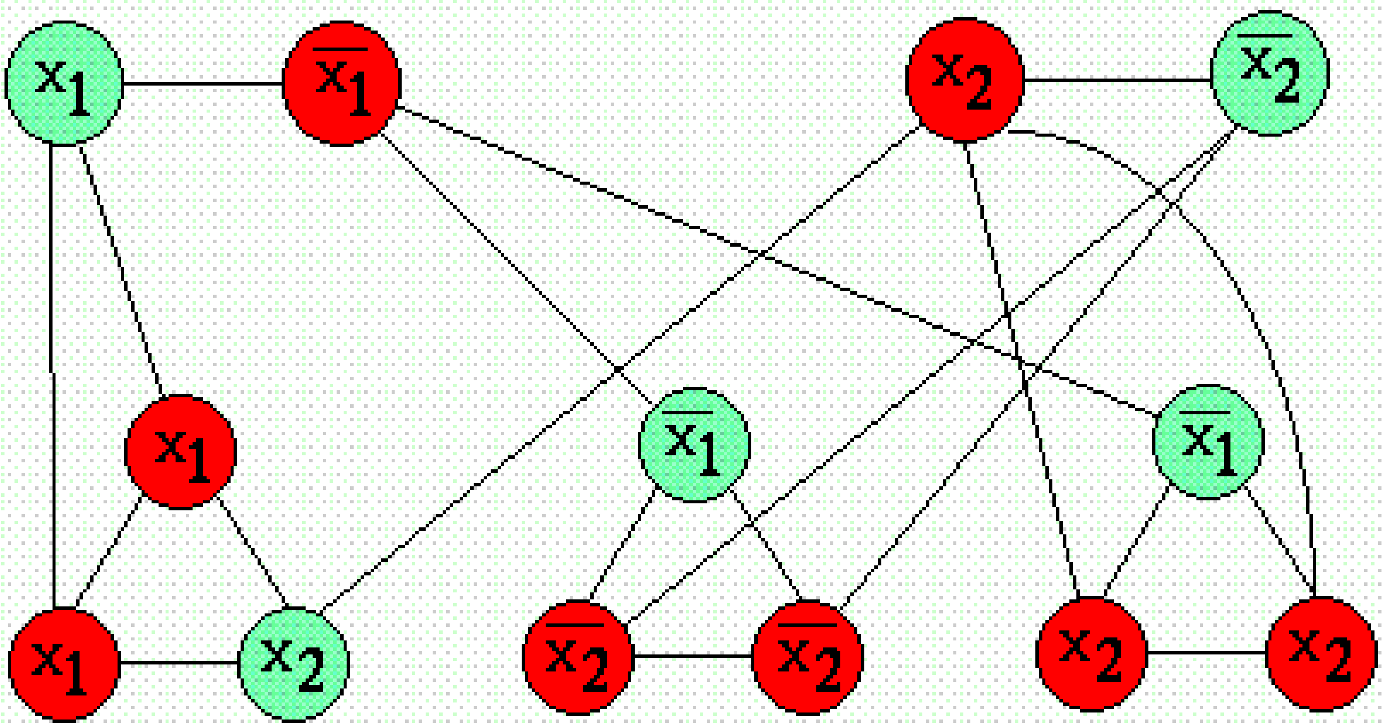
Number of clauses: m

When is there a vertex cover of order n + 2m?

Put vertices corresponding to true variables in the vertex cover.

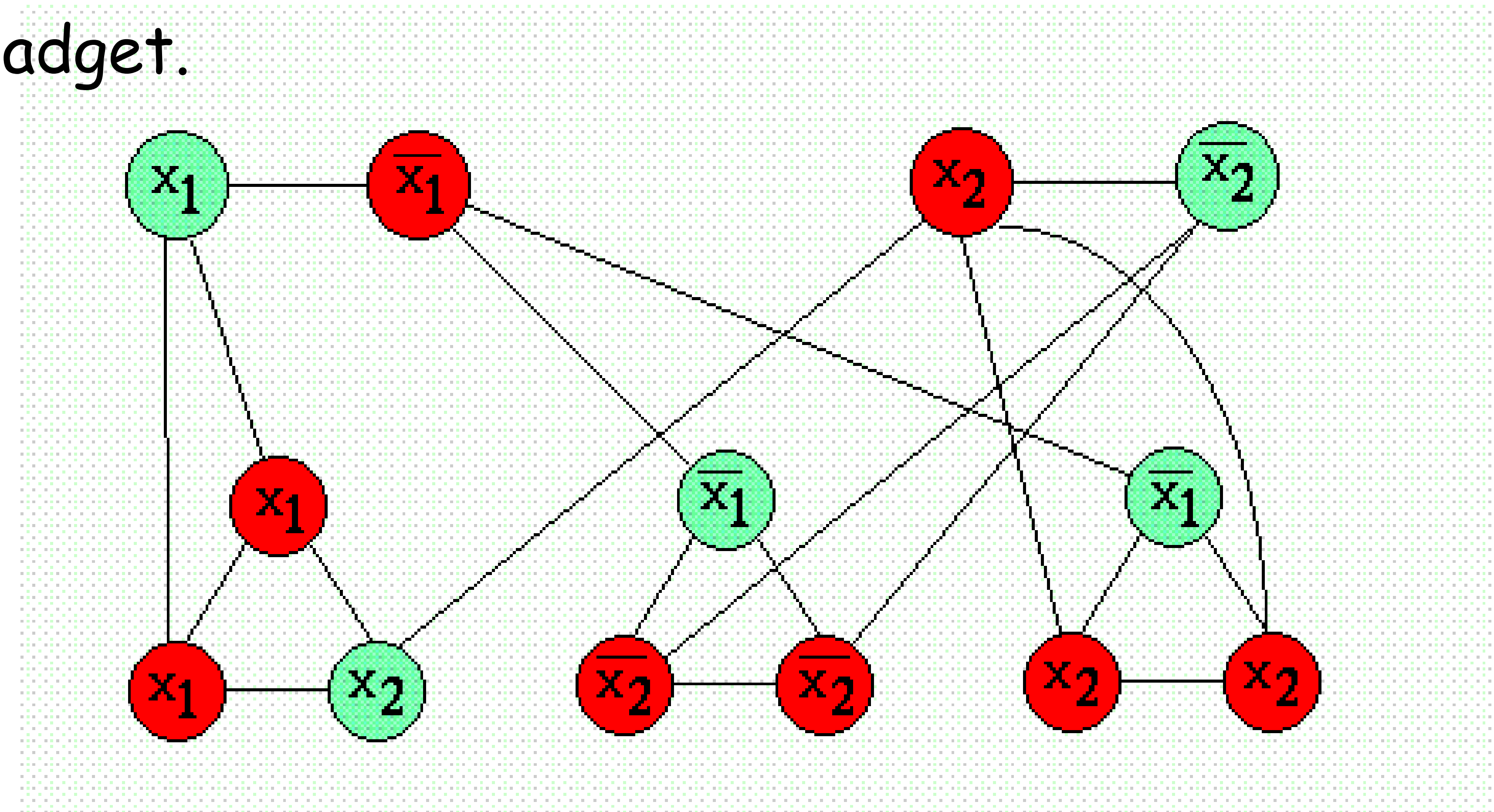


Satisfying assignment: Each clause has at least one true variable. Put two other vertices into the vertex cover:



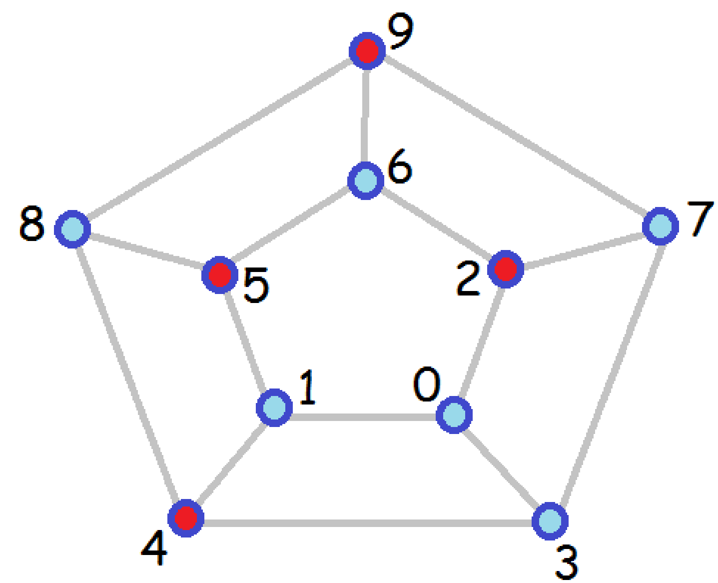
So each truth assignment corresponds to a vertex cover of order n + 2m.

Any vertex cover of order n + 2m corresponds to a satisfying assignment because we can only select at most one of x and ¬x (these are the true variables). The true variables must satisfy each clause since at most 2 vertices can be selected from each clause gadget.



INDEPENDENT SET: Given G,k, does G have an independent set of order k?

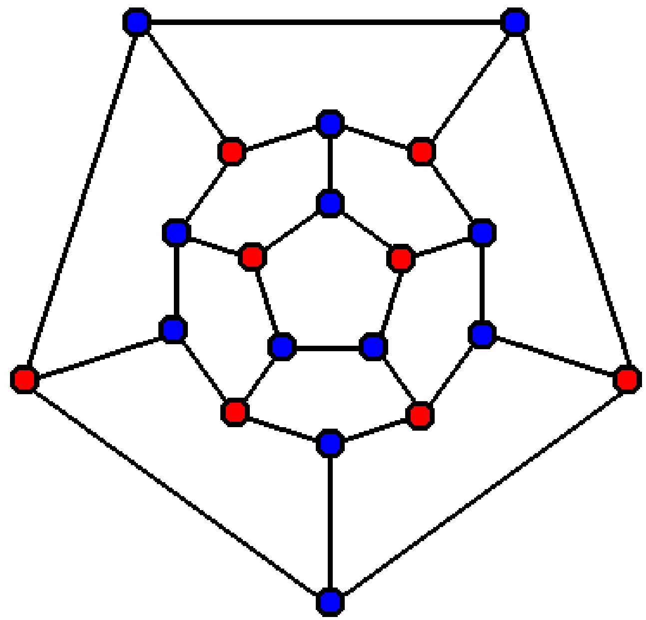
Theorem: IND. SET is NP-complete.

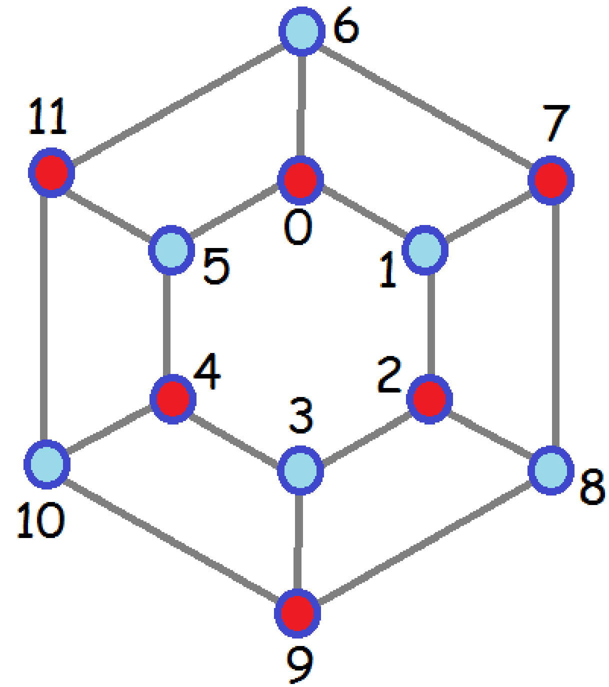


Proof: Certificate:

vertex numbers of vertices in ind. set.

To check: Time $O(k^2)$ to make sure edges are missing between each pair of vertices in the independent set.

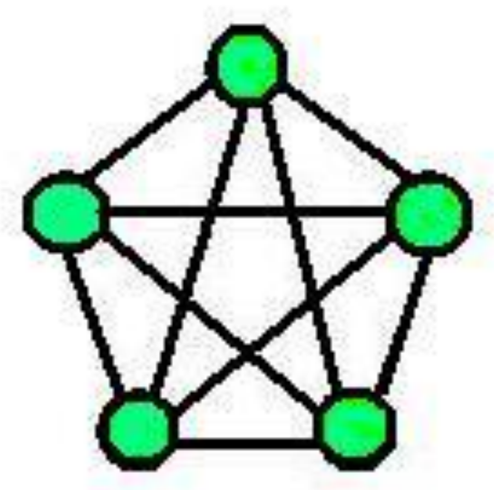




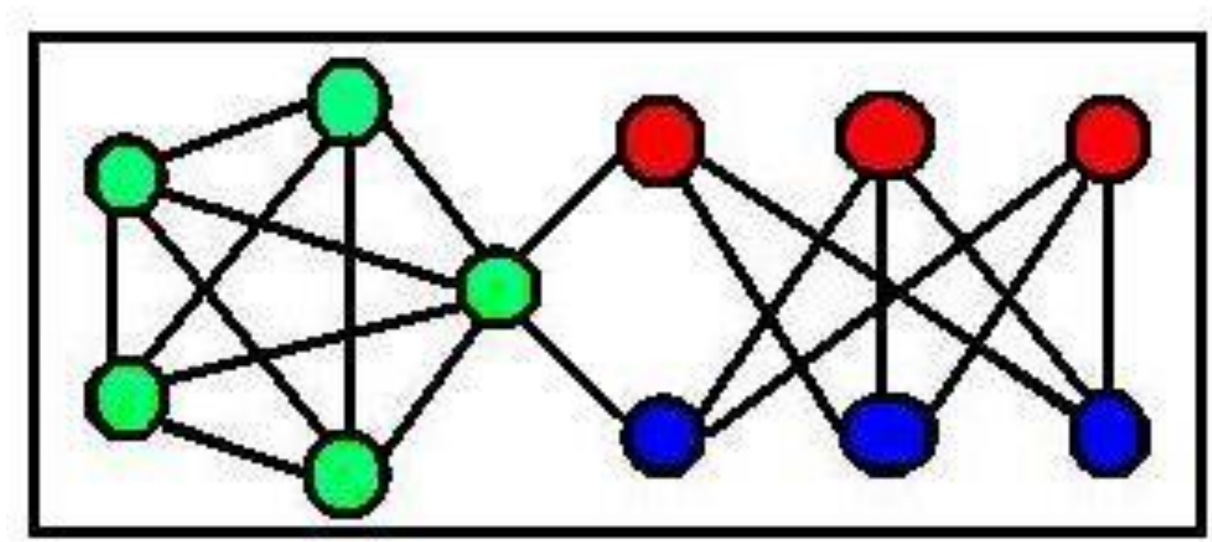
To determine if G has a vertex cover of order k: ask if G has an independent set of order n-k. The complement of every vertex cover is an independent set.

CLIQUE: Given G,k, does G have a clique of order k?

Theorem: CLIQUE is NP-complete.

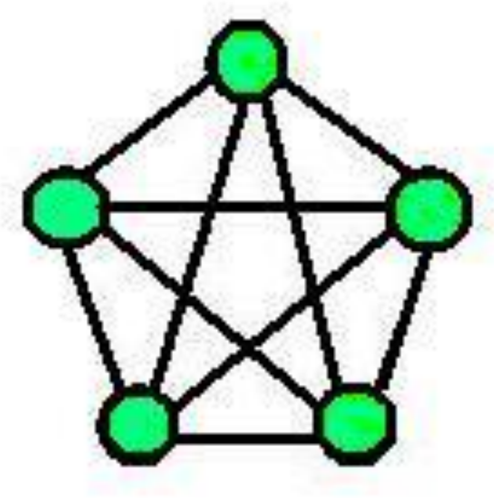


Clique of order 5.



Graph which has a clique of order 5.

The complement of G: has an edge (u, v) for each pair of vertices in G which are not connected by an edge in G.



To solve IND. SET using CLIQUE: create the complement of G in $O(n^2)$ time and then ask if it has a clique of order k.