Problem of the Day:

Factor

$(ab)^k \; c^k$

as xyz in all ways such that $y \neq \varepsilon$

and $|xy| \leq 2k$.

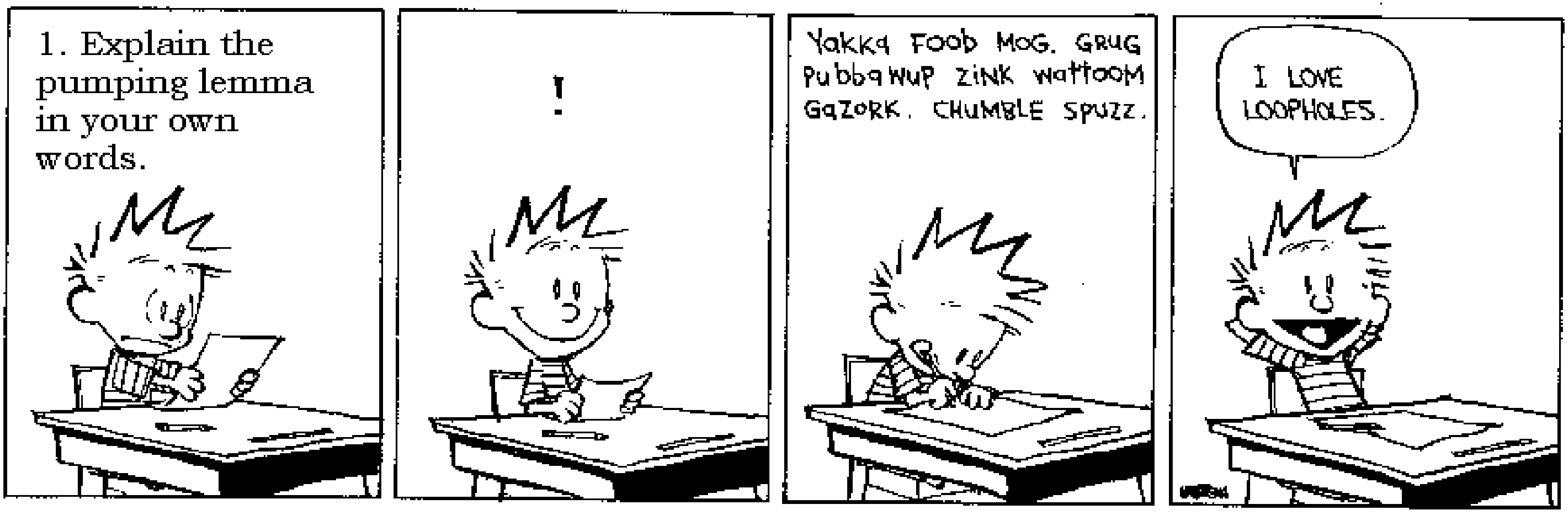
Assignment #3 has been posted.

No tutorial today.

The midterm is in class on Wednesday June 21.

Bring your schedule to class on Wednesday for June 17, 18, 19, 20 so we can pick a time for a tutorial.

Pigeons and the Pumping Lemma



$L = \{ a^n b^n : n \geq 0 \}$ is not a regular language.

Proof (by contradiction)

Assume L is regular.

Then L is accepted by a DFA M with k states for some integer k.

Since L is accepted by a DFA M with k states, the pumping lemma holds.

Let $w = a^r b^r$ where $r = \lceil k/2 \rceil$.

Consider all possibilities for y:

Case 1: $a^{i}\ (a^{j})\ a^{r-i-j}\ b^{r}$ $\quad j \geq 1$.

Pump zero times: $a^{r-j}\ b^{r} \notin L$ since $r-j < r$.

Case 2: $a^{r-i}\ (\ a^{i}\ \ b^{j}\ )\ \ b^{r-j}$ $\quad i, j \geq 1$.

Pump 2 times: $a^{r-i}\ a^{i}\ \ b^{j}\ \ a^{i}\ \ b^{j}\ \ b^{r-j} \notin L$
because it is not of the form $a^{*}\ b^{*}$

Case 3: $a^{r}\ b^{i}\ \ (b^{j})\ b^{r-i-j}$ $\quad j \geq 1$.

Pump zero times: $a^{r}\ b^{r-j} \notin$ since $r-j < r$.

Therefore, L is not regular.

L= { a^n b^n : $n \geq 0$ } is not a regular language.

Proof (by contradiction) Assume L is regular. Then L is accepted by a DFA M with k states for some integer k. Since L is accepted by a DFA M with k states, the pumping lemma holds. Let $w = a^k b^k$ (a more judicious choice for w).

Since w is in L and $|w| \geq k$, $\exists$ x, y, z such that

1. $w = x y z$,
2. $y \neq \varepsilon$,
3. $| x y | \leq k$, and
4. $x y^n z$ is in L for all $n \geq 0$.

For $w = a^k b^k$ (a more judicious choice for w):

Consider all possibilities for y:

Case 1: $a^i \ (a^j) \ a^{k-i-j} \ b^k$ $\qquad j \geq 1$.

Pump zero times: $a^{k-j} b^k \notin L$ since $k-j < k$.

This covers all cases with $|xy| \leq k$.

Therefore, L is not regular.

Using closure properties:

$L = \{w \in \{a, b\}^*$: w has the same number of a's as b's$\}$ is not regular.

Proof (by contradiction)

Assume L is regular. The language $a^* b^*$ is regular since it has a regular expression. Because regular languages are closed under intersection, $L \cap a^* b^*$ is regular. But

$L \cap a^* b^* = \{ a^n b^n : n \geq 0\}$ which is not regular.

Therefore, L is not regular.

Question from 2003 Midterm:

Apply the pumping lemma to $w = a^s\ b\ a^{s\hat{}4}$

to prove that $L = \{ a^n\ b\ a^r : n^2 \le r \le n^4 \}$

is not regular. All you may assume is that $s^4 + s + 1 \ge k$ where k is the number of states.

What is a more judicious choice for w and how does this change the proof?

Given the DFA below which accepts L, prove that $L^R = \{ u^R : u \in L\}$ is regular by designing a NDFA which accepts L^R.

For example, abaa is in L so $(abaa)^R$ = aaba is in L^R.