# CSC 320 Midterm Exam

## June 23, 2010

**Instructions:**

1. Put your name on every page of the exam.
2. No calculators or other aids. Closed book.
3. Read through the entire exam before beginning. You should have 7 pages including this header page.

| **Question** | **Value** | **Mark** |
|---|---|---|
| 1 | 30 | |
| 2 | 25 | |
| 3 | 25 | |
| 4 | 20 | |
| **Total** | **100** | |

**Name:** ___________________________________

**ID Number:** ______________________________

1.(a) [10 marks] Prove that the language

$L = \{w \in \{0, 1\}^* : w$ has $0\,1$ as a prefix and $1\,0$ as a suffix $\}$

is regular by designing a DFA which accepts $L$.

(b) [10 marks] Prove that the language

$L = \{w \in \{a, b\}^* : w$ contains both $a\,b\,a$ and $b\,a\,a\,b$ as substrings $\}$

is regular by giving a regular expression which generates $L$.

(c) [10 marks] Design a context-free grammar which generates the language

$L = \{c^r\ w\ c^s\ w^R\ c^t : w \in \{a, b\}^*, r, t \geq 1 \text{ and } s \geq 0\}.$

2. [25 marks] Use the construction described in class (which is the same as the one in the text) to convert this NDFA to an equivalent DFA:

| State | Symbol | Q | Next state |
|---|---|---|---|
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Start state: ______________________________________

Final states: ____________________________________

A picture of your final DFA:

3.(a) [5 marks] State the pumping lemma for regular languages.

(b) [5 marks] Let $w = a^r\, b\, a^r$. Describe all possible ways of choosing $x, y, z$ such that $w = x\, y\, z$, and $y \neq \varepsilon$.

(c) [10 marks] Apply the pumping lemma to $w = a^r\, b\, a^r$ to prove that $L = \{a^n\, b\, a^m : n \le m \le 8n\}$ is not accepted by a DFA with $2r + 1$ states.

(d) [5 marks] A more judicious choice for $w$ would have made the argument for (c) much simpler. Suggest a better choice for $w$. How does this simplify the argument you gave for (c)?

4. Circle **True** or **False** and justify your answer. **No marks will be given unless there is a correct justification.**

(a) [5 marks] It is possible to find a language $L$ that satifies conditions of the pumping lemma but $L$ is not regular.

True False

(b) [5 marks] Solving (finding a match) a correspondance system over the alphabet $\{a , b\}$ is easier than trying to solve a correspondance system defined over the alphabet $\{a, b, c, d\}$.

True False

(c) [5 marks] The language $L^*$ is an infinite language for all languages $L$.

True False

(d) [5 marks] Given a graph $G$, if $G - v$ has a Hamilton path for all vertices $v$, then $G$ has a Hamilton cycle.

True False

Use this page if you need more space.

Clearly indicate the question you are answering.