Use the pumping theorem for context-free languages to prove that

$L = \{ a^n b a^n b a^p \ : n, p \geq 0 , p \geq n \}$

is not context-free.

Hint: For the pumping theorem, factor in all ways as $u\ v\ x\ y\ z$ such that $|v| + |y| \geq 1$ and for each case, argue that $u\ v^n\ x\ y^n\ z$ is not in L for some n.

Announcements

Assignment #4 has been posted: due Wed. July 18.

Final exam tutorial:

Monday Aug. 6, 10am, ECS 116.

If the building is locked, I will prop open the back door to ECS (the one that opens on to the campus).

The Pumping Theorem for Context-Free Languages:

Let G be a context-free grammar.

Then there exists some constant k which depends on G such that for any string w which is generated by G with $|w| \geq k$,

there exists u, v, x, y, z, such that

1. $w = u\,v\,x\,y\,z$,
2. $|v| + |y| \geq 1$, and
3. $u\,v^n\,x\,y^n\,z$ is in L for all $n \geq 0$.

Closure Properties:

We have constructions that use the context-free grammars proving that contex-free languages are closed under union, concatenation and Kleene star.

We have a construction that uses a PDA and a DFA that proves that context-free languages are closed under intersection with regular languages.

Intersection:

$L_1 = \{ a^n b^n c^p \ : n, p \geq 0\}$

$L_2 = \{ a^n b^p c^n \ : n, p \geq 0\}$

What is $L_1 \cap L_2$?

Exclusive OR:

$L_1 = a^* \, b^* \, c^*$

$L_2 = \{ a^p b^q c^r \; : p \neq q \text{ or } p \neq r \text{ or } q \neq r \}$

What is $L_1 \oplus L_2$?

Difference:

$L_1 = a^* \, b^* \, c^*$

$L_2 = \{ a^p \, b^q \, c^r \; : p \neq q \text{ or } p \neq r \text{ or } q \neq r \}$

What is $L_1 - L_2$?

$L = \{ a^n b^n c^n \ : n \geq 0\}$

$L_1 = \{ a^p b^q c^r \ : p \neq q \}$

$L_2 = \{ a^p b^q c^r \ : p \neq r \}$

$L_3 = \{ a^p b^q c^r \ : q \neq r \}$

The complement of L is NOT equal to

$L_1 \cup L_2 \cup L_3$.

Which strings are in the complement of L but not in $L_1 \cup L_2 \cup L_3$?

$L = \{ a^n b^n c^n \ : n \geq 0\}$

$L_1 = \{ a^p b^q c^r \ : p \neq q \}$

$L_2 = \{ a^p b^q c^r \ : p \neq r \}$

$L_3 = \{ a^p b^q c^r \ : q \neq r \}$

$L_4 = \{ w \in \{a, b, c\}^* : w \notin a^* b^* c^*\}$

$L_1 \cup L_2 \cup L_3 \cup L_4$ is context-free and is the complement of L.

Therefore, context-free languages are not closed under complement.

Theorem:

$L = \{ a^p : p$ is a prime number$\}$ is not context-free.

Theorem:

$L = \{ a^{n^2} : n \geq 0\}$ is not context-free.

Note: if L is a language defined over a one symbol alphabet and you can prove L is not regular using the pumping lemma, then it also means that L is not context-free.

Theorem:

$L = \{ a^p : p$ is a prime number$\}$ is not context-free.

Assume L is context-free and is generated by a context-free grammar G. Then there exists some constant k dependent on G such that for all strings w in L of length at least k the Pumping Theorem holds. Choose $w = a^p$ for some prime number $p \geq k$.

Factor a^p in all ways as u v x y z:

$w = a^i\, a^j\, a^r\, a^s\, a^t$ where $p = i + j + r + s + t$,

$j + s \geq 1$, $v = a^j$ and $y = a^s$.

Pumping n+1 times yields:

$a^i \; (a^j)^{n+1} \; a^r \; (a^s)^{n+1} \; a^t = a^i\, a^r\, a^t\, (a^j)^{n+1} (a^s)^{n+1}$

$= a^{p-j-s + (j+s)(n+1)}$

Let $x = j+s$.

Pumping n+1 times yields: a^{p+xn}

$w = a^i a^j a^r a^s a^t$

Let $x = j+s$.

Pumping n+1 times yields: a^{p+xn}

Pump p+1 times:

Gives $a^{p+xp} = a^{p(x+1)}$

Since $x \geq 1$, $(x+1) \geq 2$ and so $p(x+1)$

Cannot be prime.

Theorem:

$L = \{ a^{n^2} : n \geq 0\}$ is not context-free.

Note: if L is a language defined over a one symbol alphabet and you can prove L is not regular using the pumping lemma, then it also means that L is not context-free.