

$$L = \{ a^p b^q c^r d^s : p, q, r, s \geq 0, p+q = r+s \}$$

1. Design a PDA that accepts L .

Hint: use four states, one for reading each different type of symbol.

2. Design a context-free grammar that generates L .

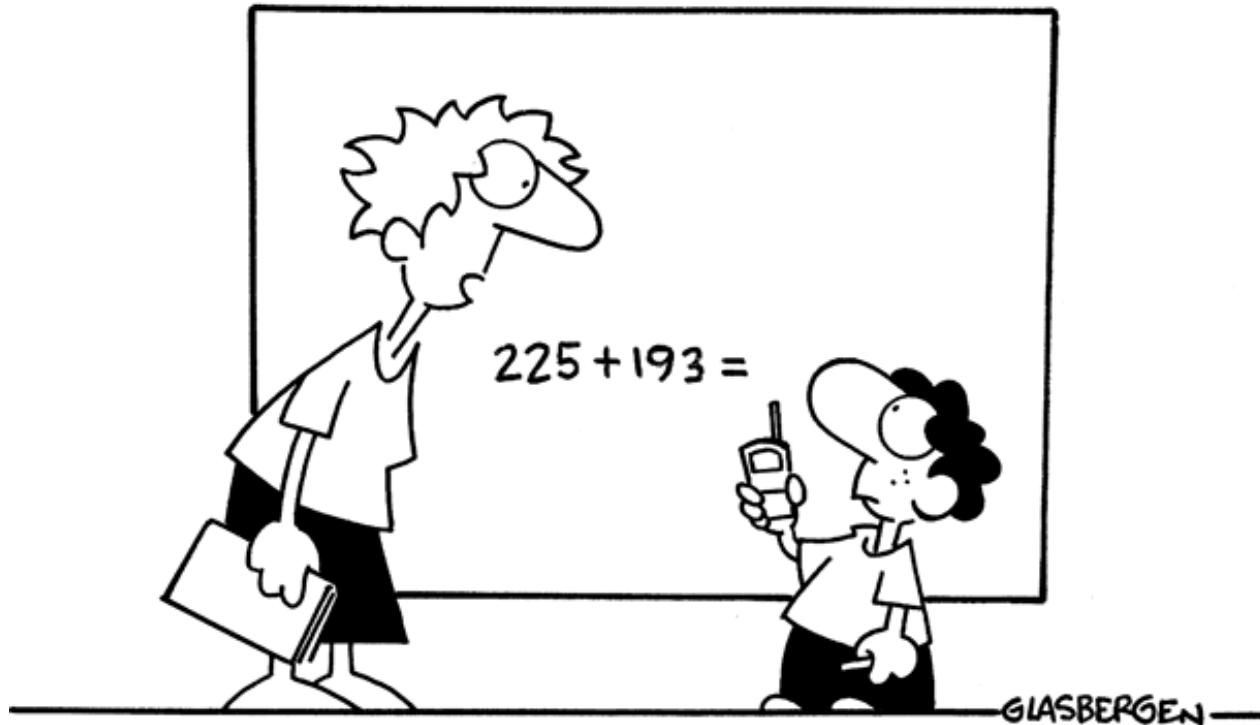
Midterm tutorial: Monday June 25 at 7:30pm, **ECS 116**.

Assignment #3 is due at the beginning of class on Tuesday June 26.

No class on Friday: I hope you can attend some of the Turing Celebration events in its place.

To get maximum benefit from homework and old exams, solve problems yourself instead of doing a web search for solutions.

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"You have to solve this problem by yourself. You can't call tech support."

A grammar for L_2 :

$$L = \{ a^p b^q c^r d^s : p, q, r, s \geq 0, p+q = r+s \}$$

Meaning:

S: match a with d

T: match a with c

U: match b with d

V: match b with c

$$L = \{ a^p b^q c^r d^s : p, q, r, s \geq 0, p+q = r+s \}$$

Meaning:

S: match a with d

T: match a with c

U: match b with d

V: match b with c

Start symbol: S

$$S \rightarrow a S d$$

$$S \rightarrow T$$

$$S \rightarrow U$$

$$S \rightarrow V$$

$$T \rightarrow a T c$$

$$T \rightarrow V$$

$$U \rightarrow b U d$$

$$U \rightarrow V$$

$$V \rightarrow b V c$$

$$V \rightarrow \varepsilon$$

Design a PDA which accepts the language
 $\{ u u^R v v^R : u, v \in \{a,b\}^+ \}$

Hint: A bottom of the stack symbol could prove useful in helping to ensure that u matches with u^R and v matches with v^R .

Languages which are context-free.

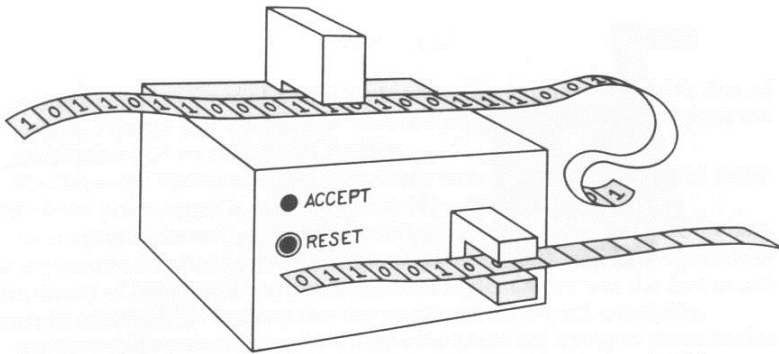
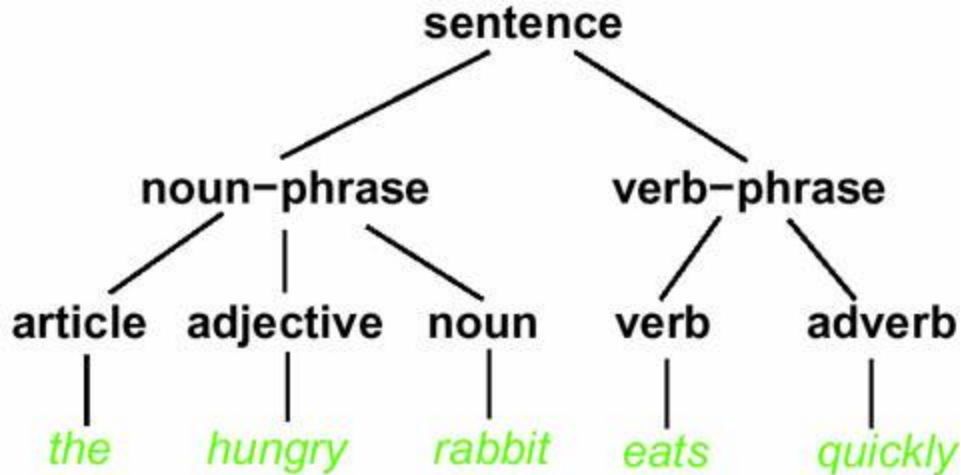
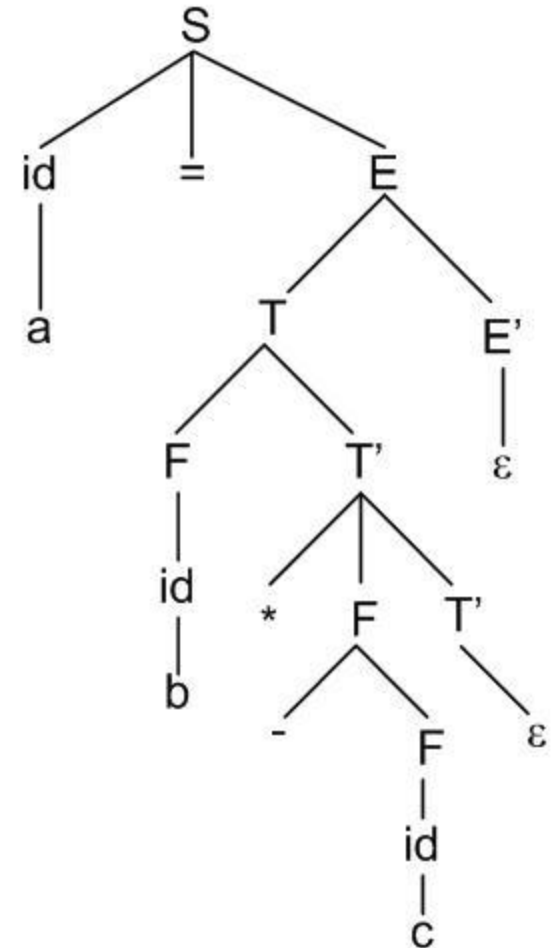


Figure 7.4 A push-down automaton



A Derivation Tree



Theorem:

If L is $L(G)$ for some context-free grammar G , then there is a PDA M which accepts L .

Proof: By construction of a PDA which mimics derivations in the grammar.

A context-free grammar, start symbol S :

$$S \rightarrow \varepsilon$$

$$S \rightarrow A B A$$

$$A \rightarrow aa$$

$$B \rightarrow b S a$$

Apply construction to get a PDA.

Theorem: If L is a language which is accepted by some PDA M , then there is a context-free grammar which generates L .

Proof: The basic idea of the proof is to first define simple PDA's (except for at the start, one symbol is popped and 0,1, or 2 symbols are pushed for each transition).

They then show how to construct a context-free grammar from a simple PDA.

You are not responsible for this proof.

Closure Properties of Context-free Languages

Intersecting a context-free language and a regular language gives a context-free language.

Context-free languages are closed under union, concatenation and Kleene star.

Context-free languages are not closed under intersection or complement.

Thought question: are they closed under difference/exclusive or?

Theorem:

If L_1 is context-free and L_2 is regular then $L_1 \cap L_2$ is context-free.

Proof:

By construction. This proof is similar to the one on the assignment proving closure of regular languages under intersection.

$$L = \{ w c w^R : w \in \{a, b\}^* \} \cap a^* c a^*$$

Start state: s , Final State: $\{t\}$

State	Input	Pop	Next state	Push
s	a	ϵ	s	A
s	b	ϵ	s	B
s	c	ϵ	t	ϵ
t	a	A	t	ϵ
t	b	B	t	ϵ

Theorem:

Context-free languages are closed under union, concatenation and Kleene star.

Proof: By construction.

Let $G_1 = (V_1, \Sigma, R_1, S_1)$ and

let $G_2 = (V_2, \Sigma, R_2, S_2)$.

We show how to construct a grammar $G = (V, \Sigma, R, S)$ for $L(G_1) \cup L(G_2)$, $L(G_1) \cdot L(G_2)$, and $L(G_1)^*$.

$$L_1 = \{ a^n b^{2n} : n \geq 0 \}$$

$$S_1 \rightarrow a S_1 b b$$

$$S_1 \rightarrow \varepsilon$$

$$L_2 = \{ u u^R v : u, v \in \{a, b\}^+ \}$$

$$S_2 \rightarrow U_2 V_2$$

$$U_2 \rightarrow a U_2 a$$

$$V_2 \rightarrow a V_2$$

$$U_2 \rightarrow b U_2 b$$

$$V_2 \rightarrow b V_2$$

$$U_2 \rightarrow aa$$

$$V_2 \rightarrow a$$

$$U_2 \rightarrow bb$$

$$V_2 \rightarrow b$$

$$L_1 = \{ a^n b^{2n} : n \geq 0 \}$$

$$S_1 \rightarrow a S_1 b b$$

$$S_1 \rightarrow \varepsilon$$

UNION:

Start symbol S

$$S \rightarrow S_1 \quad S \rightarrow S_2$$

$$L_2 = \{ u u^R v : u, v \in \{a, b\}^+ \}$$

$$S_2 \rightarrow U_2 V_2 \quad U_2 \rightarrow a U_2 a \quad V_2 \rightarrow a V_2$$

$$U_2 \rightarrow b U_2 b \quad V_2 \rightarrow b V_2$$

$$U_2 \rightarrow aa \quad V_2 \rightarrow a$$

$$U_2 \rightarrow bb \quad V_2 \rightarrow b$$

$$L_1 = \{ a^n b^{2n} : n \geq 0 \}$$

$$S_1 \rightarrow a S_1 b b$$

$$S_1 \rightarrow \varepsilon$$

CONCATENATION:

Start symbol S

$$S \rightarrow S_1 S_2$$

$$L_2 = \{ u u^R v : u, v \in \{a, b\}^+ \}$$

$$S_2 \rightarrow U_2 V_2 \quad U_2 \rightarrow a U_2 a \quad V_2 \rightarrow a V_2$$

$$U_2 \rightarrow b U_2 b \quad V_2 \rightarrow b V_2$$

$$U_2 \rightarrow aa \quad V_2 \rightarrow a$$

$$U_2 \rightarrow bb \quad V_2 \rightarrow b$$

KLEENE STAR:

Start symbol S

$$S \rightarrow S_1 S$$

$$S \rightarrow \varepsilon$$

$$L_1 = \{ a^n b^{2n} : n \geq 0 \}$$

$$S_1 \rightarrow a S_1 b b$$

$$S_1 \rightarrow \varepsilon$$

KLEENE STAR: Start symbol S

$$S \rightarrow S_2 S$$

$$S \rightarrow \varepsilon$$

$$L_2 = \{ u u^R v : u, v \text{ in } \{a, b\}^+ \}$$

$$S_2 \rightarrow U_2 V_2$$

$$U_2 \rightarrow a U_2 a$$

$$V_2 \rightarrow a V_2$$

$$U_2 \rightarrow b U_2 b$$

$$V_2 \rightarrow b V_2$$

$$U_2 \rightarrow aa$$

$$V_2 \rightarrow a$$

$$U_2 \rightarrow bb$$

$$V_2 \rightarrow b$$

Theorem: $L = \{ a^n b^n c^n : n \geq 0 \}$ is not context-free.

Proof: Next class using the pumping theorem for context-free languages.

Today: assume it is true.

$$L_1 = \{ a^p b^q c^r : p = q \}$$

Design a context-free grammar for L_1 .

$$L_2 = \{ a^p b^q c^r : p = r \}$$

Design a PDA for L_2 .

This proves L_1 and L_2 are context-free.

$$L_1 \cap L_2 = \{ a^n b^n c^n : n \geq 0 \}.$$

Therefore, context-free languages are not closed under intersection.

$$L = \{ a^n b^n c^n : n \geq 0 \}$$

$$L_1 = \{ a^p b^q c^r : p \neq q \}$$

$$L_2 = \{ a^p b^q c^r : p \neq r \}$$

$$L_3 = \{ a^p b^q c^r : q \neq r \}$$

The complement of L is NOT equal to
 $L_1 \cup L_2 \cup L_3$.

Which strings are in the complement of L but not in $L_1 \cup L_2 \cup L_3$?

$$L = \{ a^n b^n c^n : n \geq 0 \}$$

$$L_1 = \{ a^p b^q c^r : p \neq q \}$$

$$L_2 = \{ a^p b^q c^r : p \neq r \}$$

$$L_3 = \{ a^p b^q c^r : q \neq r \}$$

$$L_4 = \{ w \in \{a, b, c\}^* : w \notin a^* b^* c^* \}$$

$L_1 \cup L_2 \cup L_3 \cup L_4$ is context-free and is the complement of L .

Therefore, context-free languages are not closed under complement.