Problem of the Day:

Factor $(ab)^k$ as xyz in all ways such that $y \neq \varepsilon$.

Assignment #2 is due on Friday.

Midterm: Friday June 27 in class.

Old midterms are available from our class web page. Yesterday I added the midterm from Fall 2011.

$L = \{ a^n b^n : n \geq 0 \}$ is not a regular language.

Proof (by contradiction)

Assume L is regular.

Then L is accepted by a DFA M with k states for some integer k.

Since L is accepted by a DFA M with k states, the pumping lemma holds.

Let $w = a^r b^r$ where $r = \lceil k/2 \rceil$.

Consider all possibilities for y:

Case 1: $a^i\ (a^j)\ a^{r-i-j}\ b^r$ $\quad j \geq 1.$

Pump zero times: $a^{r-j}\ b^r \notin L$ since $r-j < r$.

Case 2: $a^{r-i}\ (a^i\ b^j)\ b^{r-j}$ $\quad i, j \geq 1.$

Pump 2 times: $a^{r-i} a^i\ b^j\ a^i\ b^j\ b^{r-j} \notin L$
because it is not of the form $a^* b^*$

Case 3: $a^r\ b^i\ (b^j)\ b^{r-i-j}$ $\quad j \geq 1.$

Pump zero times: $a^r\ b^{r-j} \notin$ since $r-j < r$.

Therefore, L is not regular.

$L = \{ a^n b^n : n \geq 0 \}$ is not a regular language.

Proof (by contradiction) Assume L is regular. Then L is accepted by a DFA M with k states for some integer k. Since L is accepted by a DFA M with k states, the pumping lemma holds. Let $w = a^k b^k$ (a more judicious choice for w).

Since w is in L and $|w| \geq k$, $\exists$ x, y, z such that

1. $w = x y z$,
2. $y \neq \varepsilon$,
3. $| x y | \leq k$, and
4. $x y^n z$ is in L for all $n \geq 0$.

For $w = a^k b^k$ (a more judicious choice for w):

Consider all possibilities for y:

Case 1: $a^i\ (a^j)\ a^{k-i-j}\ b^k$ $\quad j \geq 1$.

Pump zero times: $a^{k-j} b^k \notin L$ since $k-j < k$.

This covers all cases with $|xy| \leq k$.

Therefore, L is not regular.

Using closure properties:

$L = \{w \in \{a, b\}^*$: w has the same number of a's as b's$\}$ is not regular.

Proof (by contradiction)

Assume L is regular. The language $a^* b^*$ is regular since it has a regular expression. Because regular languages are closed under intersection, $L \cap a^* b^*$ is regular. But

$L \cap a^* b^* = \{ a^n b^n : n \geq 0\}$ which is not regular.

Therefore, L is not regular.

Question from 2003 Midterm:

Apply the pumping lemma to $w = a^s\, b\, a^{s\hat{}4}$

to prove that $L = \{ a^n\, b\, a^r : n^2 \le r \le n^4 \}$

is not regular. All you may assume is that $s^4 + s + 1 \ge k$ where k is the number of states.

What is a more judicious choice for w and how does this change the proof?

Given the DFA below which accepts L, prove that $L^R = \{ u^R : u \in L\}$ is regular by designing a NDFA which accepts L^R.

For example, abaa is in L so $(abaa)^R$ = aaba is in L^R.