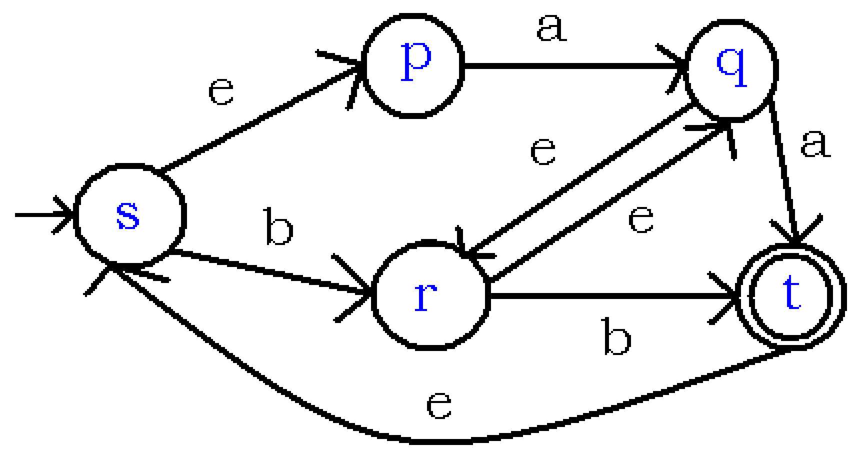
Convert to a DFA:

Start state:

Final States:



State	Symbol	Read- Q	E(Q)
	a		
	b		
	a		
	b		
	a		
	b		

Announcements

Assignment #2: Due at beginning of class Friday June 8.

Read through the assignment and take any questions you have about it to the tutorial next week.

$P = \{(a, bb)(a, ab), (b, ca), (ca, a),$

$(abc, c), (cccc, acc)\},$

MATCH:

$(a, ab)\ (b, ca)\ (ca, a)\ (a, ab)\ (abc, c)$

$a\ \ b\ \ ca\ \ a\ \ abc\ \ = abcaaabc$

$ab\ \ ca\ \ a\ \ ab\ \ c\ \ = abcaaabc$

CSC 320 Lecture 10: DFA's and Regular Expressions

(Comic by Randall Munroe.)

Lecture 11: Regular Languages

The goal of the lecture is to prove that:

R= { L : L is generated by a regular expression}, and

S= { L : L is L(M) for a DFA(M)}

are the same sets of languages (R=S).

We already proved that S is the same set as

T= {L : L = L(M) for some NDFA M} (last class).

Before proving this, it is helpful to first show that the set S is closed under union, concatenation and Kleene star.

A set S is closed with respect to a binary operation · if for all s and t in S, s · t is in S.

Examples:

N= {0, 1, 2, 3, ...}

Closed for addition and multiplication.

Not closed under subtraction or division.

The set $S = \{ L : L$ is $L(M)$ for some DFA $M\}$ is closed under union.

Let $M_1 = (K_1, \Sigma, \delta_1, s_1, F_1)$ accept L_1

and $M_2 = (K_2, \Sigma, \delta_2, s_2, F_2)$ accept L_2.

Proof 1: A construction for a new DFA $M = (K, \Sigma, \delta, s, F)$ which accepts $L_1 \cup L_2$.

Proof 2: A construction for a new NDFA $M = (K, \Sigma, \Delta, s, F)$ which accepts $L_1 \cup L_2$.

The set $S=\{ L : L \text{ is } L(M) \text{ for some DFA } M\}$ is closed under concatenation.

Let $M_1= (K_1, \Sigma, \delta_1, s_1, F_1)$ accept L_1

and $M_2= (K_2, \Sigma, \delta_2, s_2, F_2)$ accept L_2.

Proof: A construction for a new NDFA $M= (K, \Sigma, \Delta, s, F)$ which accepts $L_1 \cdot L_2$.

The set S= { L : L is L(M) for some DFA M} is closed under Kleene star.

Let $M_1 = (K_1, \Sigma, \delta_1, s_1, F_1)$ accept L_1.

Proof: A construction for a new NDFA $M = (K, \Sigma, \Delta, s, F)$ which accepts $L_1{}^*$.

Theorem:

If L is a regular language, then L is L(M) for some DFA M.

Proof:

By showing how to construct a NDFA for L.

Last lecture, we proved by construction that for every NDFA, there is an equivalent DFA.

So this indirectly gives a construction for a DFA.

Regular expressions over Σ:

[Basis] 1. Φ and σ for each $\sigma \in \Sigma$ are regular expressions.

[Inductive step] If α and β are regular expressions, then so are:

2. $(\alpha\beta)$

3. $(\alpha \cup \beta)$ and

4. α^*

Note: Regular expressions are strings over

$\Sigma \cup \{ \, (\, , \,) \, , \, \Phi \, , \cup \, , \, * \, \}$

for some alphabet Σ.

Theorem: If L is accepted by a DFA M, then there is a regular expression which generates L.

There is a proof which constructs a regular expression from the DFA in the text (in the proof of Theorem 2.3.2). I expect you to know that this theorem is true but you are not responsible for the proof.

Conclusion: A language is regular if and only if it is accepted by a finite automaton.

The set of S= { L : L is L(M) for some DFA M} is closed under complement.

Let M_1= (K_1, Σ, δ_1, s_1, F_1) accept L_1.

Proof: A construction for a new DFA M= (K, Σ, δ, s, F) which accepts the complement of L_1.

Regular languages are also closed for intersection (assignment #2).