

Proof of the Day:

The **length** of a string is the number of symbols it contains. For example, the length of **01101** is five.

Let $L = \{ w \text{ such that } w \text{ is a string over the alphabet } \{0,1\} \text{ and the last symbol in } w \text{ is a } 0 \}$.

- (a) Give an inductive definition of the strings in L .
- (b) Write down all of the strings of L that have length at most four.
- (c) How many strings in L have length k ?
Express your answer as a formula.
- (d) Prove your formula is correct by induction.

Office hours:

TWF: 11:30-1:30.

I will start by answering questions here after our class and will go to my office to wait if you need to see me at 12:30 onwards.

BUT: please let me know either by e-mail the day before or talking to me before the end of class what time you would like to come by so I don't have to hang around on days nobody wants to see me. Or you can send me e-mail or make an appointment for help.

If you have classes at these times, please make an appointment (preferably for TWF).

Don't forget to sign the attendance sheet every class you attend.

Do NOT sign the sheet if you do not plan on staying for the class.

Do NOT sign the sheet for anybody else.

Do NOT ask someone else to sign the sheet for you.

It will waste class time if I am forced to police the attendance sheet.

CSC 320:

To Infinity and Beyond

Copyright 2002 by Randy Glasbergen.
www.glasbergen.com

BOOKS



**"Yes, we have *Chicken Soup for the Math Teacher's Soul*.
The price is $\$475 \div 23 \times .018^2 - Y^3 + 4X \div \$73.99999 + 2$."**



Outline

- Definitions of equinumerous, cardinality, finite, infinite, countable, uncountable
- Tactics for proving that sets are countable
- Diagonalization for proving sets are uncountable

CSC 320 will challenge you to broaden how you think mathematically.

The idea that there is more than one “size” of infinity is a strange concept. Several new proof tactics are introduced.

On ongoing theme of this class will be that each of our language representation schemes represents a countable number of languages so some must be left out since the total number of possible languages is not countable.

Definitions:

Two sets A and B are **equinumerous** if there is a bijection (pairing) $f:A \rightarrow B$

A set S is **finite** (**cardinality n**) if it is equinumerous with $\{1, 2, \dots, n\}$ for some integer n .

A set is **infinite** if it is not finite.

A set is **countably infinite** if it is equinumerous with the set of natural numbers.

A set is **countable** if it is finite or countably infinite.

Theorem 1:

The set $\{(p, q): p \text{ and } q \text{ are natural numbers}\}$ is countable

Proof tactic: Dovetailing

Note: this tactic does not provide an explicit formula for a bijection to the natural numbers but it does have enough information for someone to know how to compute it.

You should give this same level of information on the assignment.

Theorem 2:

The set $\{ r : r \text{ is a real number, } 0 \leq r < 1 \}$
is not countable.

Proof tactic: Diagonalization

Note: Use diagonalization on the assignment
when you want to prove a set is not countable.
Do not use other results.