

CSC 225: Proof of the Day

Prove by induction that:

$$\sum_{i=0}^k 2^i = 2^{k+1} - 1$$



Put your name on your answer and hand in your proof.

All attempts will be given full participation marks
(correct or not).

Announcements

Note: The midterm is scheduled on **Wed. June 27** so that I can hand it back marked before the drop deadline.

Any questions about the course outline?

Assignment #1 and Tutorial #1 are posted.

Tutorials start next week.

Bring your schedule to class on Friday (to help me in choosing office hours).

Starting this term, the standard Faculty of Engineering grade conversions have been changed to a new university-wide standard:

Letter Grade	OLD: Engineering	NEW: UVic
A+	90-100	same
A	85-89	same
A-	80-84	same
B+	75-79	77-79
B	70-74	73-76
B-	65-69	70-72
C+	60-64	65-69
C	55-59	60-64
D	50-54	50-59

Consequences: it will be harder to make B and C grades.
More students will have D grades.

WARNING: D grades jeopardize your COOP status.

How to get an A+ in CSC 320

With 23 points:

A+	87	
A	80.5	
A-	74	
B+	67.5	70
B	61	65
B-	54.5	61
C+	48	54.5
C	41.5	48
D	35	

With 50 points:

A+	80	
A	70	
A-	60	
B+	50	54
B	40	46
B-	30	40
C+	20	30
C	10	20
D	0	

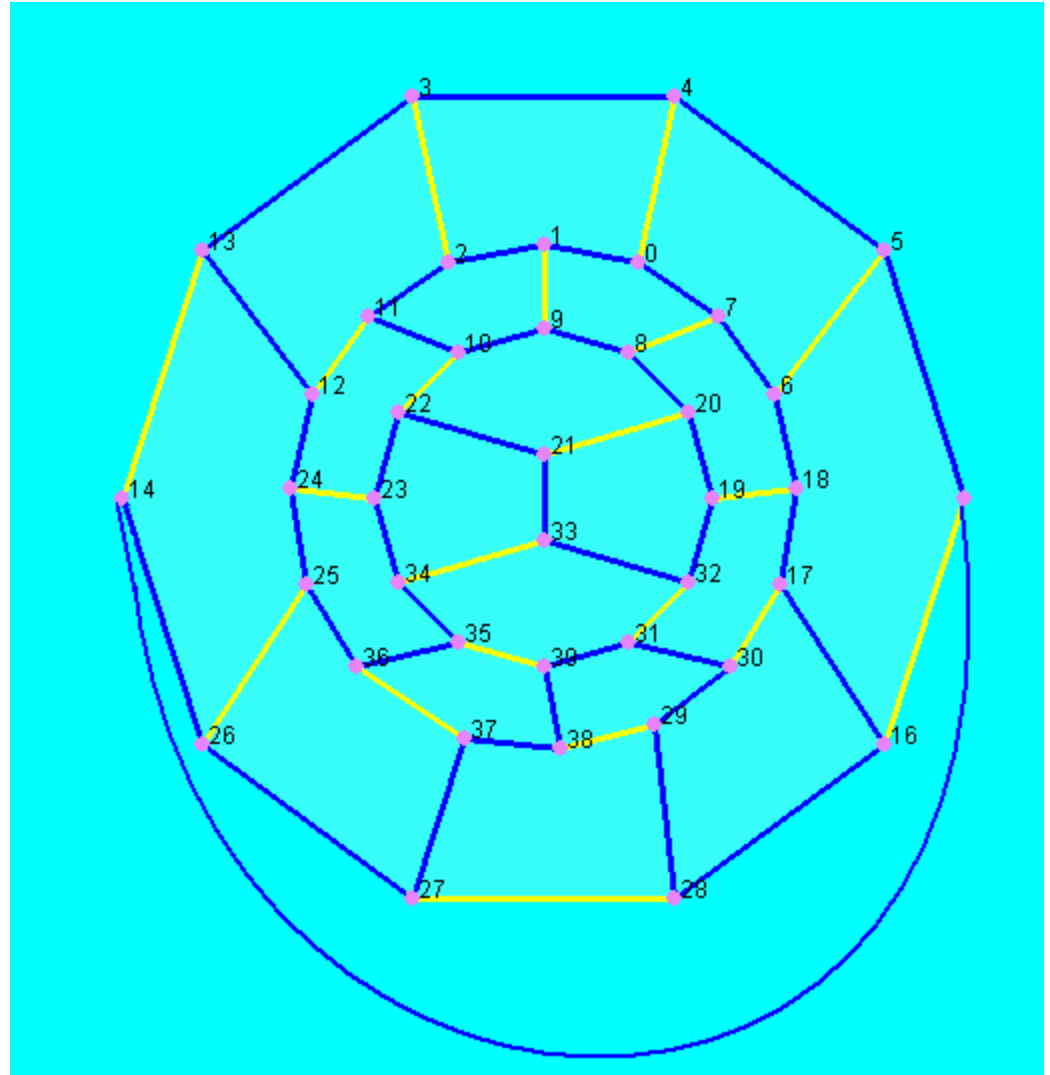
RED: New UVic Grading scheme.

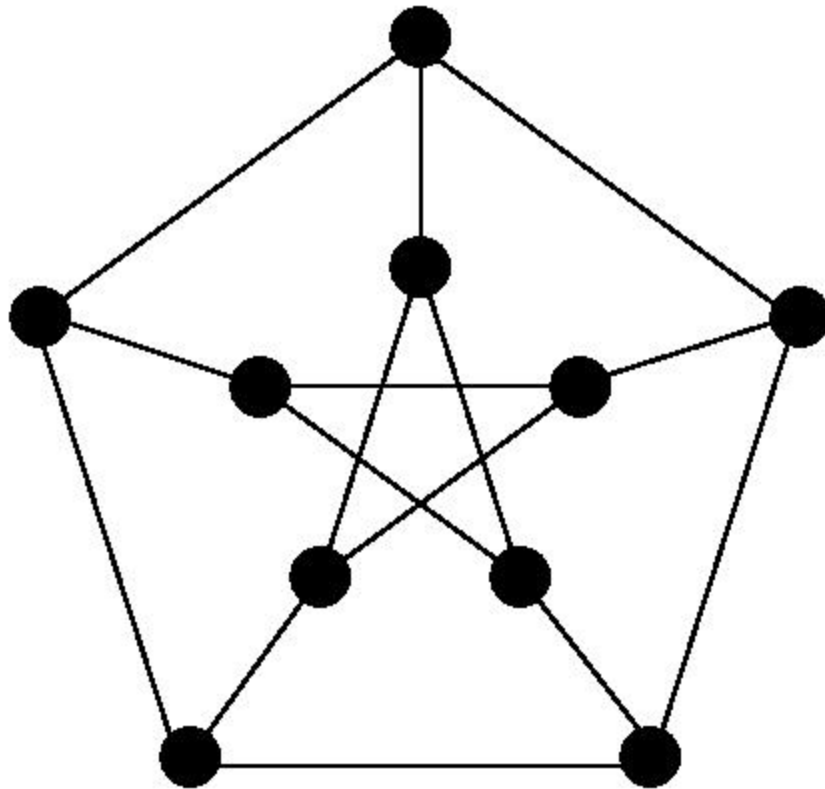
Hamilton Cycles

A cycle which includes all the vertices of a graph.

Fullerenes: 3-regular planar graphs, face sizes 5 and 6.

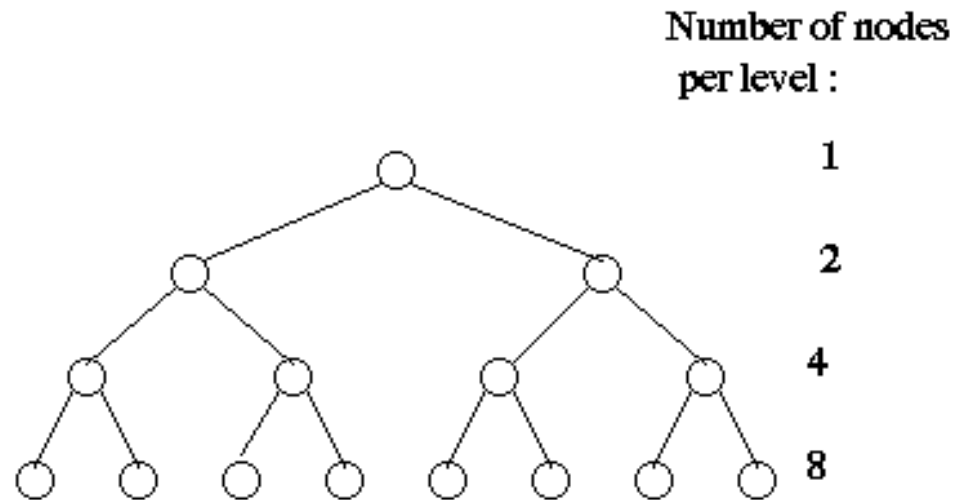
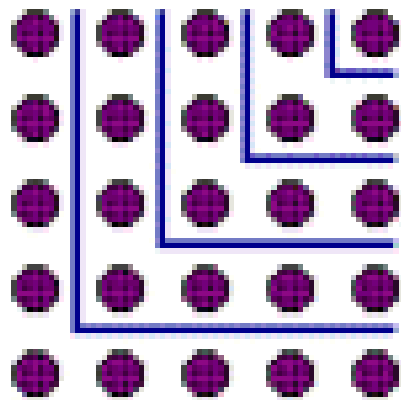
Conjecture: Every fullerene has at least one Hamilton cycle.





The Petersen
graph has no
Hamilton cycles.

Review of Induction



$$n^2 = 1 + 3 + 5 + \dots + (2n - 1)$$

Overview

- Questions from last class
- Review of induction

Induction is very similar to recursion and one goal of this class is to become skilled at writing recursive programs.

It is a useful tool for proving that programs are correct.

Time complexities of algorithms will be computed by solving recurrence relations. Induction can then be used to prove that the answers you find are correct.

Natural Numbers

$$\mathbb{N} = \{ 0, 1, 2, 3, 4, \dots \}$$

Inductive Definition:

[Basis] 0 is in the set $\mathbb{N}$

[Inductive step]:

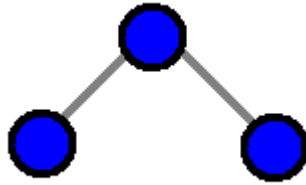
If k is in $\mathbb{N}$

then $k+1$ is in $\mathbb{N}$

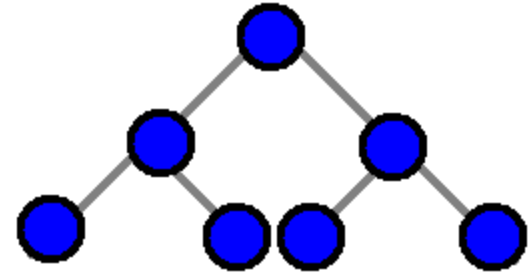
Complete Binary Trees:



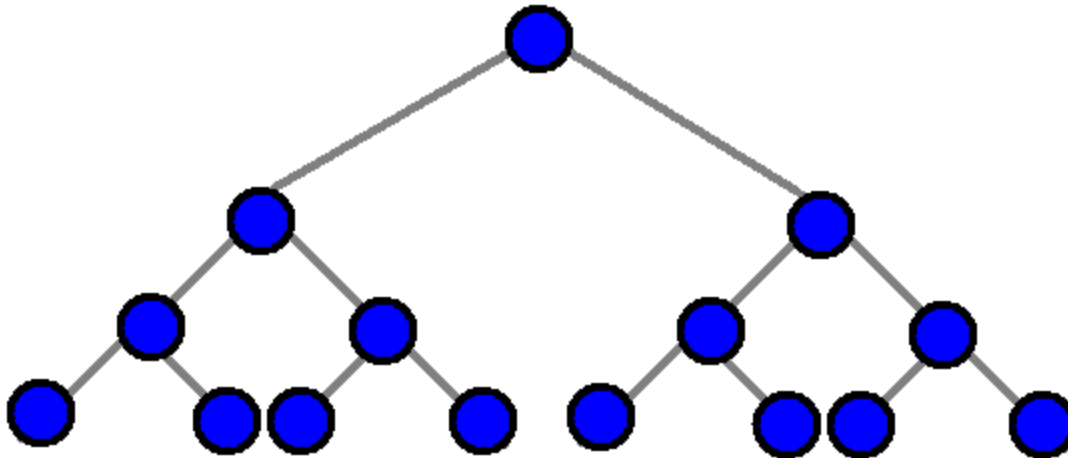
Height 0



Height 1

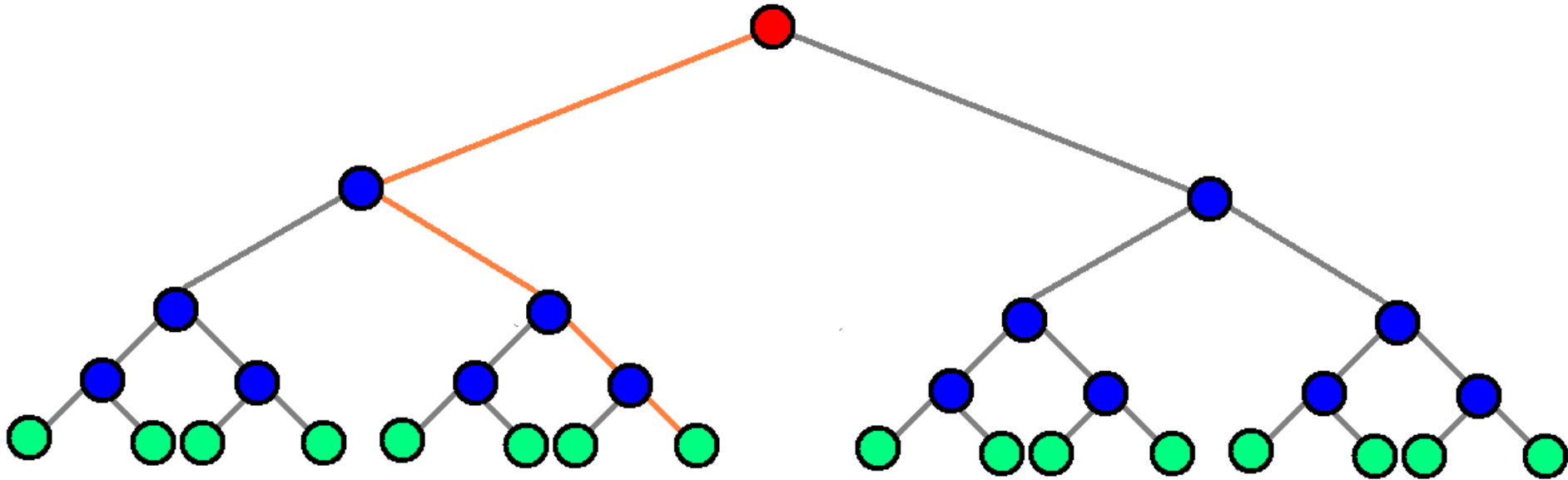


Height 2



Height 3

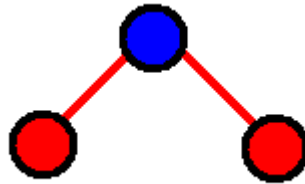
root vertex



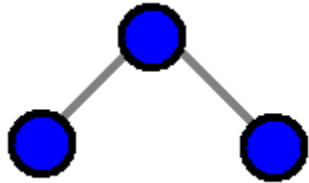
Leaves: vertices with one incident edge

Height: maximum distance from root to a leaf measured by number of edges on the path.

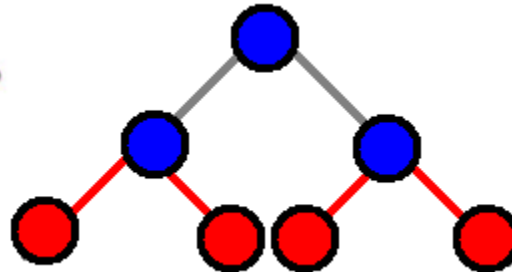
Height 0



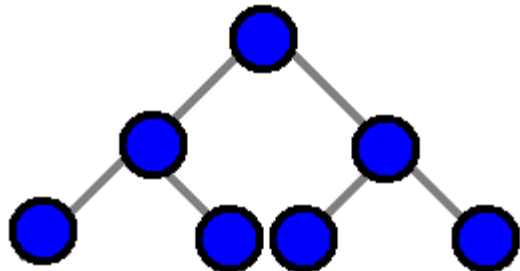
Height 1



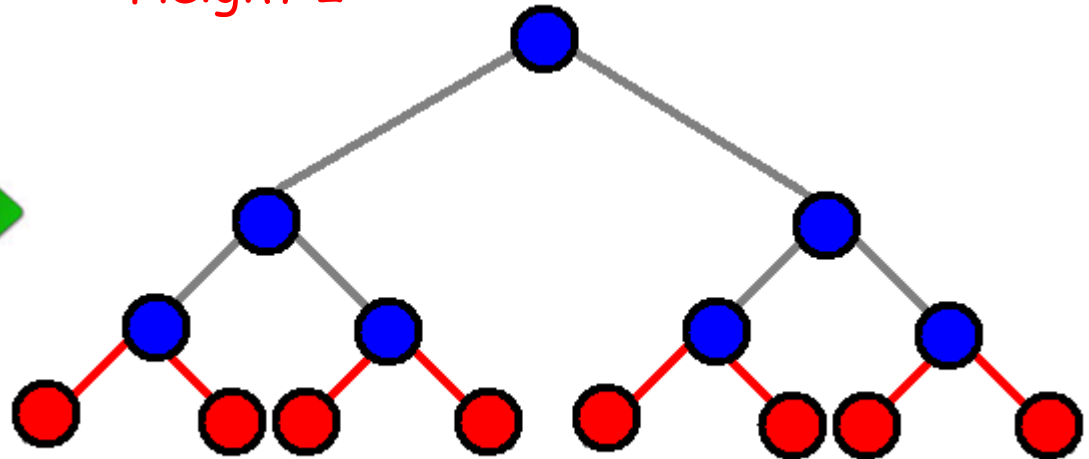
Height 1



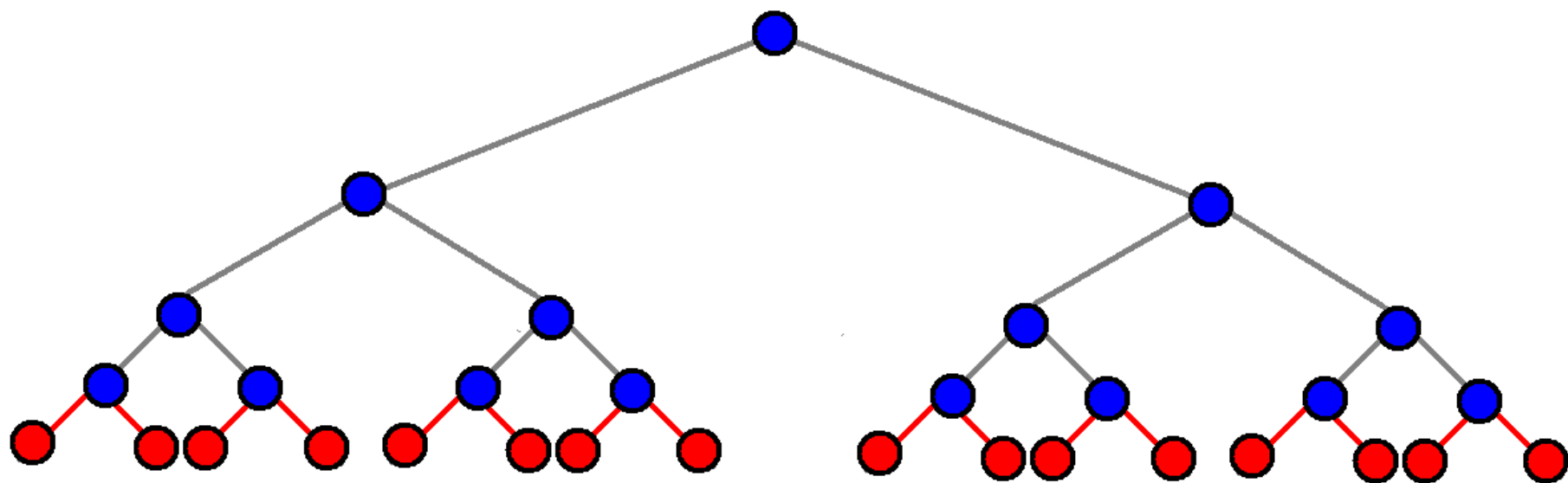
Height 2



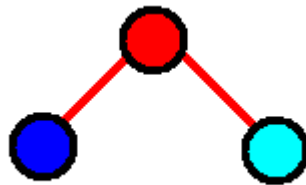
Height 2



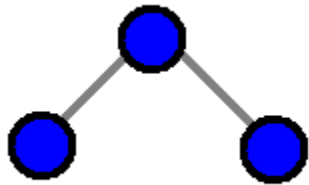
Height 3



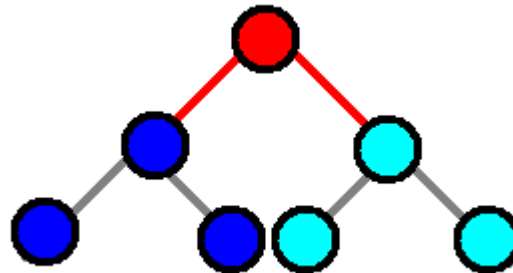
Height 0



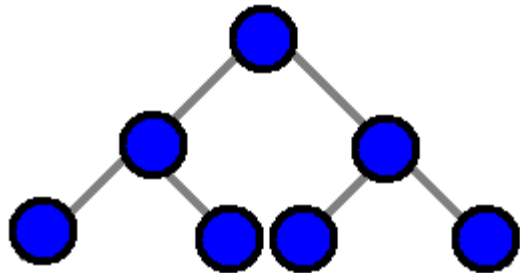
Height 1



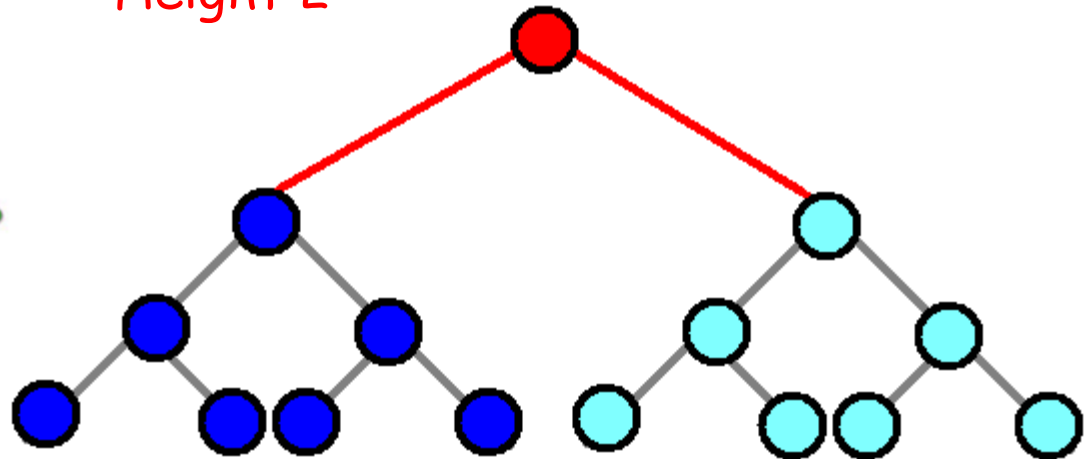
Height 1



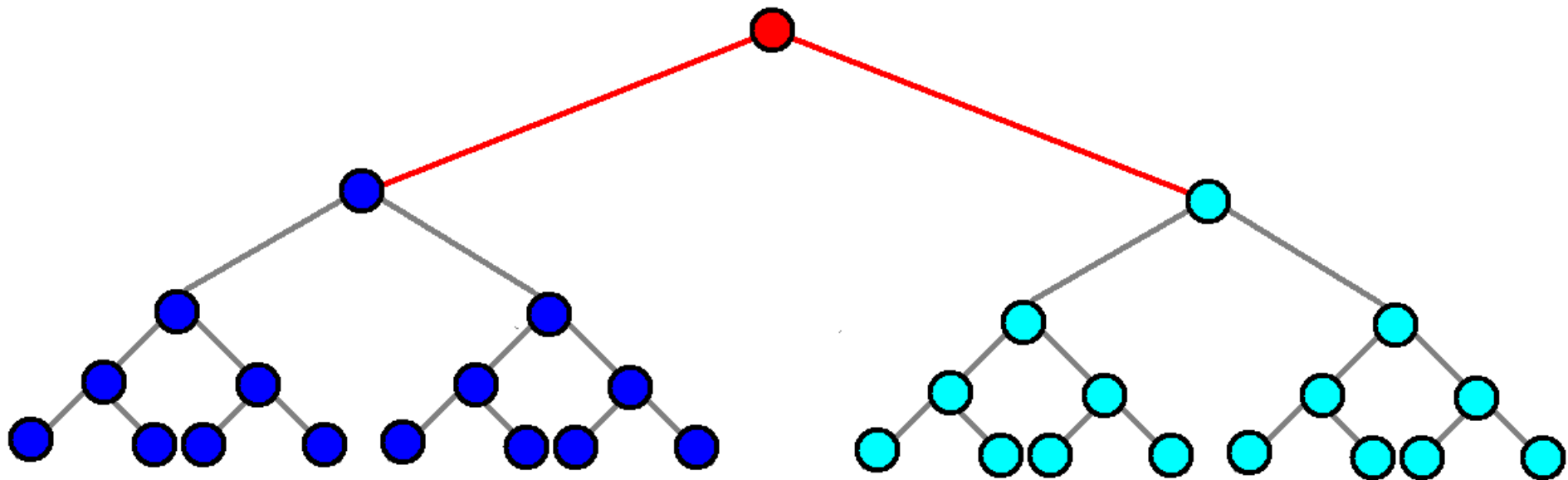
Height 2

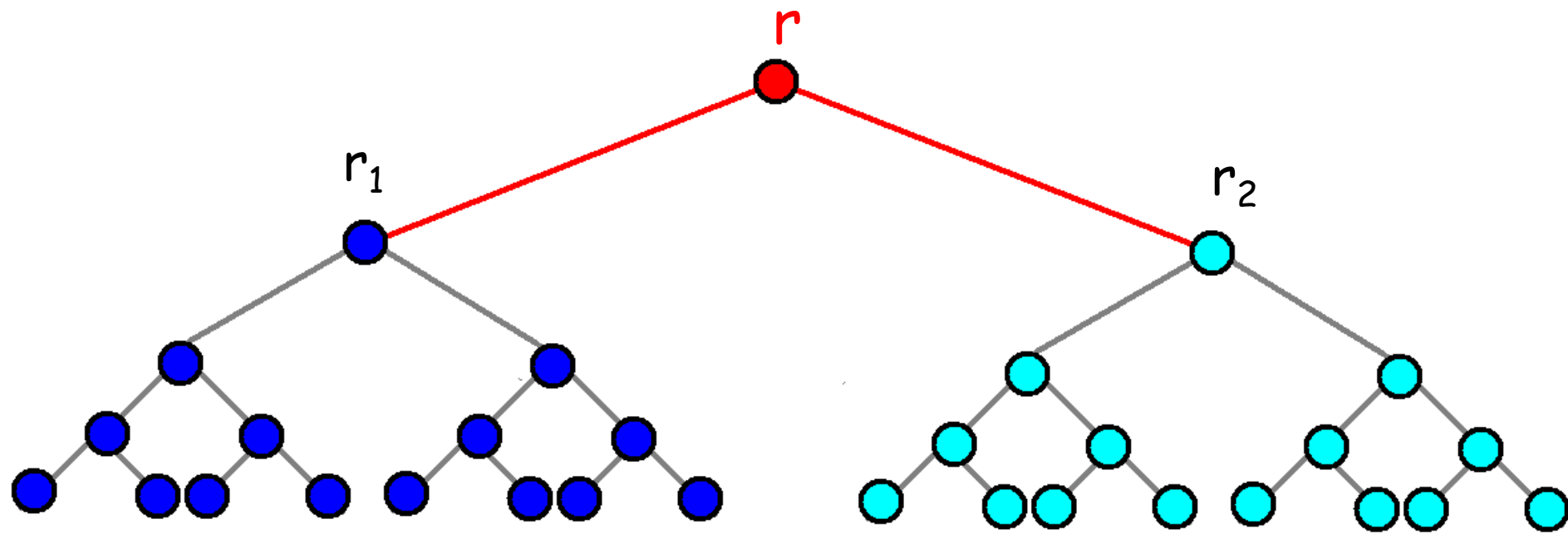


Height 2



Height 3





How can we give an inductive definition of a complete binary tree of height h ?

How many nodes does a complete binary tree of height h have?

Prove the answer is correct by induction.

Create a recurrence relation $T(h)$ where $T(h)$ gives the number of nodes of a complete binary tree of height h .

Solve your recurrence relation and prove the answer is correct by induction.