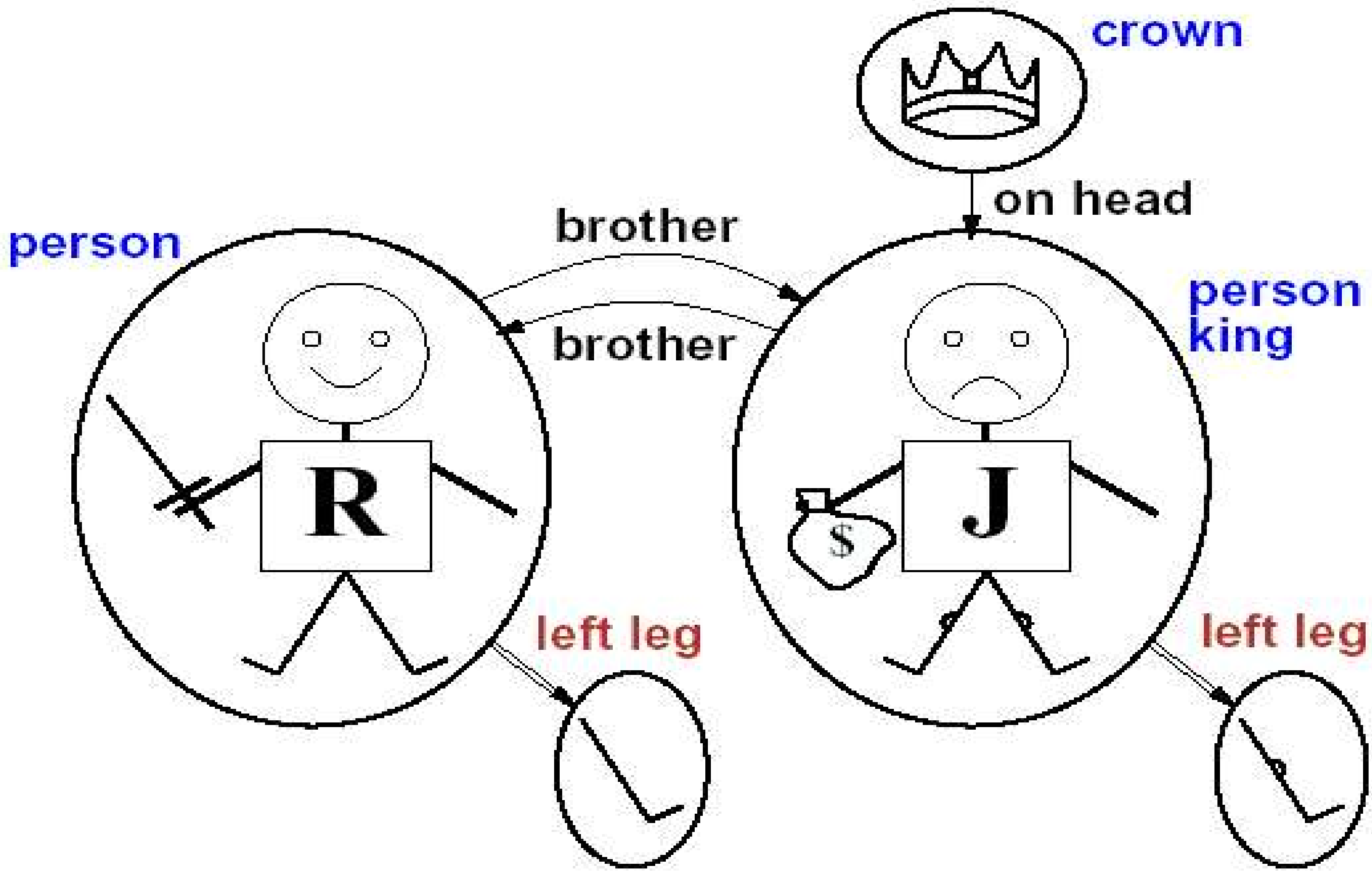
CSC421 Intro to Artificial Intelligence

UNIT 11:
First-Order Logic

Assignment discussion

- What was the biggest challenge ?
- What did you learn doing it ?
- How could it be improved ?
- What did you hate about the assignment ?
- Was it related to what we covered in class ?
- Other questions ?

Models for FOL example



Talking about collections of objects

- Variables x, y, z
- Quantifiers
  - Universal ∀
  - Existential ∃
- Expressing concisely
  - All kings are persons
  - Squares neighboring the wumpus are smelly
  - There is a piece of gold somewhere in the world

Universal Quantifier

- ∀ <variables> <sentence>
- Everyone at CSC421 is smart
  - ∀ x In(x, CSC421) => Smart(x)
  - ∀ x P is true in model m iff P is true with x being each possible object in the model
  - Roughly speaking equivalent to the conjuctions of instantiations of P
  - (In(Adam, CSC421)=>Smart(Adam))
    ∧ (In(Neesha, CSC421)=>Smart(Adam))
    ∧ (In(CSC421, CSC421) => Smart(CSC421))
    ∧ . . . .

Existential Quantification

- ∃ <variables> <sentence>
- Someone at UVic is smart
  - $\exists x\ At(x, UVic) \wedge Smart(x)$
  - $\exists x\, P$ is true in a model m iff P is true with x being some possible object in the model
  - Roughly speaking, equivalent to the disjunction of instantiations of P
  - $(At(Adam, CSC421) \wedge Smart(Adam))$
    $\vee\ (In(Neesha, CSC421) \wedge Smart(Adam))$
    $\vee\ (In(CSC421, CSC421) \wedge Smart(CSC421))$
    $\vee \ldots$

Common mistakes

- Typically, => is the main connective with ∀
- Common mistake:
  - Using ∧ instead: ∀ x In(x, CSC421) ∧ Smart(x)
  - Means everyone is in CSC421 and everyone is smart
- Typically, ∧ is the main connective with ∃
- Common mistake
  - Using => instead: ∃ x At(x, UVic) => Smart(x)
  - Is true if there is anyone who is not at UVic

Properties of quantifiers

- $\forall x \forall y$ is the same as $\forall y \forall x$
- $\exists x \exists y$ is the same as $\exists y \exists x$
- $\exists x \forall y$ is NOT the same as $\forall x \exists y$
- $\exists x \forall y$ Loves(x, y)
  - There is a person x who loves everyone y in the world
- $\forall y \exists x$ Loves(x,y)
  - Everyone y in the world is loved by at least one person x
- Quantifier duality: each can be expressed using the other

Fun with sentences

- Brothers are siblings
- “Sibling” is symmetric
- One's mother is one's female parent
- A first cousin is a child of a parent's sibling

Fun with sentences

- Brothers are siblings
  - $\forall$x,y Brother(x,y) => Sibling(x,y)
- "Sibling" is symmetric
  - $\forall$x,y Sibling(x,y) <=> Sibling(y,x)
- One's mother is one's female parent
  - $\forall$x,y Mother(x,y) <=> Female(x) ^ Parent(x,y)
- A first cousin is a child of a parent's sibling
  - $\forall$x,y Cousin(x,y) <=> $\exists$ p,ps Parent(p,x) ^ Sibling(ps,p) ^ Parent(ps, y)

Equality

- term1 = term2 is true under a given interpretation iff term1 and term2 refer to the same object
- For example definition of full sibling based on parent
  - ?

Equality

- term1 = term2 is true under a given interpretation iff term1 and term2 refer to the same object
- For example definition of full sibling based on parent
  - $\forall$ x,y Sibling(x,y) <=> [$\neg$ (x = y) $\wedge \exists$ m,f $\neg$ (m = f) $\wedge$ parent(m,x) $\wedge$ parent(f,x) $\wedge$ parent(m,y) $\wedge$ parent(f,y)]