CSC421 Intro to Artificial Intelligence

UNIT 10:
First-Order Logic

Resolution

- Sound and complete inference for propositional logic

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

Example:

$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$

Concrete example

- If there is a pit in one of [1,1], [2,2], and [3,1] and there is none in [2,2] then it is either in [1,1] or [3,1]
- KB1: $P_{1,1} \lor P_{2,2} \lor P_{3,1}$
- KB2: $\neg P_{2,2}$
- Resolution of KB1 and KB2 is $P_{1,1} \lor P_{3,1}$

Limitations of Propositional Logic

- Keeping track of location ?
- Single Wumpus ?
- Proliferation of clauses
- Doesn't deal concisely with time, space and universal patterns of relationships
- Need more expressivity – next chapter First-Order Logic

Summary

- Logical agents apply inference to a knowledge base to derive new information and make new decisions
- Basic logic concepts:
  - Syntax, semantics, Entailment, Inference, Soundness, Completeness
- Forward and Backward chaining are linear inference algorithms for Horn clauses
- Resolution is complete for propositional logic
- Propositional logic lacks expressive power

First-order Logic

- Whereas propositional logic assumes world contains facts FOL assumes the world is:
  - Objects: people, houses, theories colors, basketball games
  - Relations: red, round, boring, prime, brother of, siblings (unary relations are properties)
  - Functions: father of, best friend, one_more_than
- What is the difference between a function and a relation ?

Some examples

- One plus two equals three
- Squares neighboring the wumpus stink
- Evil King John ruled England in 1200
- The brother of Richard the Lionheart ruled England in 1200
- What are the objects, relationships, functions in these objects ?

Logics in general

Logics in general

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief
Fuzzy logic	facts + degree of truth	known interval value

How can a logic be described ?
Higher-order logics -> a relationship is transitive
Concise representations + semantics = sound reasoning procedures

Syntax of FOL

- Constants : Bach, Uvic, 2, CS421
- Predicates : Brother, >, on_top_of
- Functions : Sqrt, Left_leg_of
- Variables : x,y,z, a_1, ...
- Connectives: $\neg \wedge \vee \Rightarrow \Leftrightarrow$
- Equality : =
- Quantifiers : = $\forall \exists$

Atomic Sentences

- Atomic sentence = predicate($term_1$, ...$term_N$)
  or term1 = term2
- Term = function($term_1$, ...$term_N$)
  or constant or variable
  – Basically a term is just a complicated type of name
- For example:
  – Brother(KingJohn, RichardLionHeart)
  – GreaterThan(Length(LeftLegOf(Richard)), Length(LeftLegOf(John)))

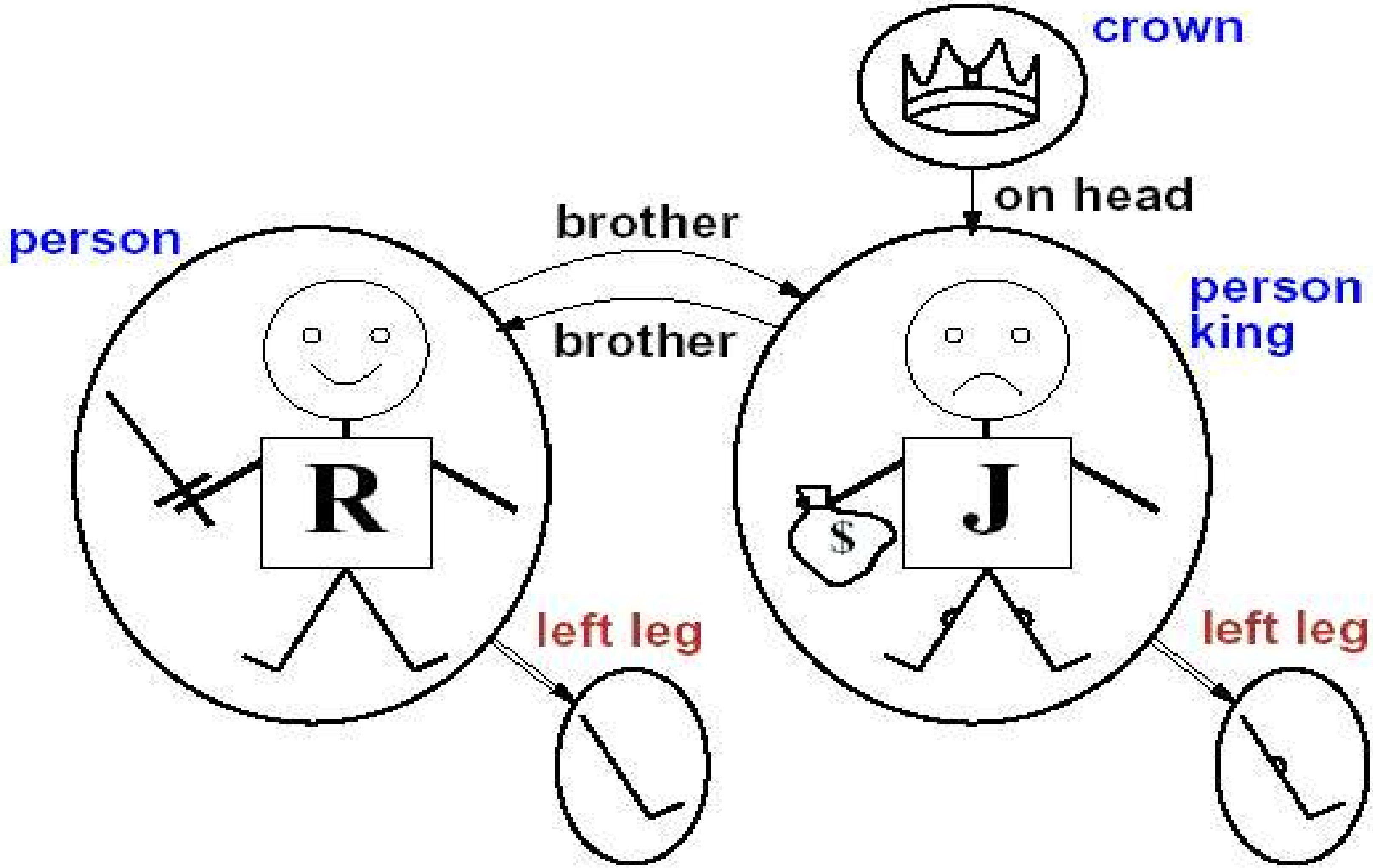
Complex Sentences

- Complexes sentences are made out of atomic sentences using connectives
- For example:
  - Sibling(Richard, John) => Sibling(John, Richard)

Truth in FOL

- Sentences are true with respect to a model and an interpretation
- Model contains objects (domain elements) and relations between them
- Interpretation specifies referents (names):
  - Constant symbols -> objects
  - Predicate symbols -> relations
  - Function symbols -> functional relations
- An atomic sentence is predicate($t_1,\ldots,t_n$) is true iff the objects referred to by $t_1,\ldots,t_n$ are in the relation referred by predicate

Models for FOL example



Talking about collections of objects

- Variables x, y, z
- Quantifiers
  - Universal ∀
  - Existential ∃
- Expressing concisely
  - All kings are persons
  - Squares neighboring the wumpus are smelly
  - There is a piece of gold somewhere in the world

Universal Quantifier

- ∀ <variables> <sentence>
- Everyone at CSC421 is smart
  - ∀ x In(x, CSC421) => Smart(x)
  - ∀ x P is true in model m iff P is true with x being each possible object in the model
  - Roughly speaking equivalent to the conjuctions of instantiations of P
  - (In(Adam, CSC421)=>Smart(Adam))
    ∧ (In(Neesha, CSC421)=>Smart(Adam))
    ∧ (In(CSC421, CSC421) => Smart(CSC421))
    ∧ . . . .

Existential Quantification

- ∃ <variables> <sentence>
- Someone at UVic is smart
  - $\exists x \; At(x, UVic) \wedge Smart(x)$
  - $\exists x \, P$ is true in a model m iff P is true with x being some possible object in the model
  - Roughly speaking, equivalent to the disjunction of instantiations of P
  - $(At(Adam, CSC421) \wedge Smart(Adam))$
    $\vee \; (In(Neesha, CSC421) \wedge Smart(Adam))$
    $\vee \; (In(CSC421, CSC421) \wedge Smart(CSC421))$
    $\vee \ldots$

Common mistakes

- Typically, => is the main connective with $\forall$
- Common mistake:
  - Using $\wedge$ instead: $\forall$ x In(x, CSC421) $\wedge$ Smart(x)
  - Means everyone is in CSC421 and everyone is smart
- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake
  - Using => instead: $\exists$ x At(x, UVic) => Smart(x)
  - Is true if there is anyone who is not at UVic