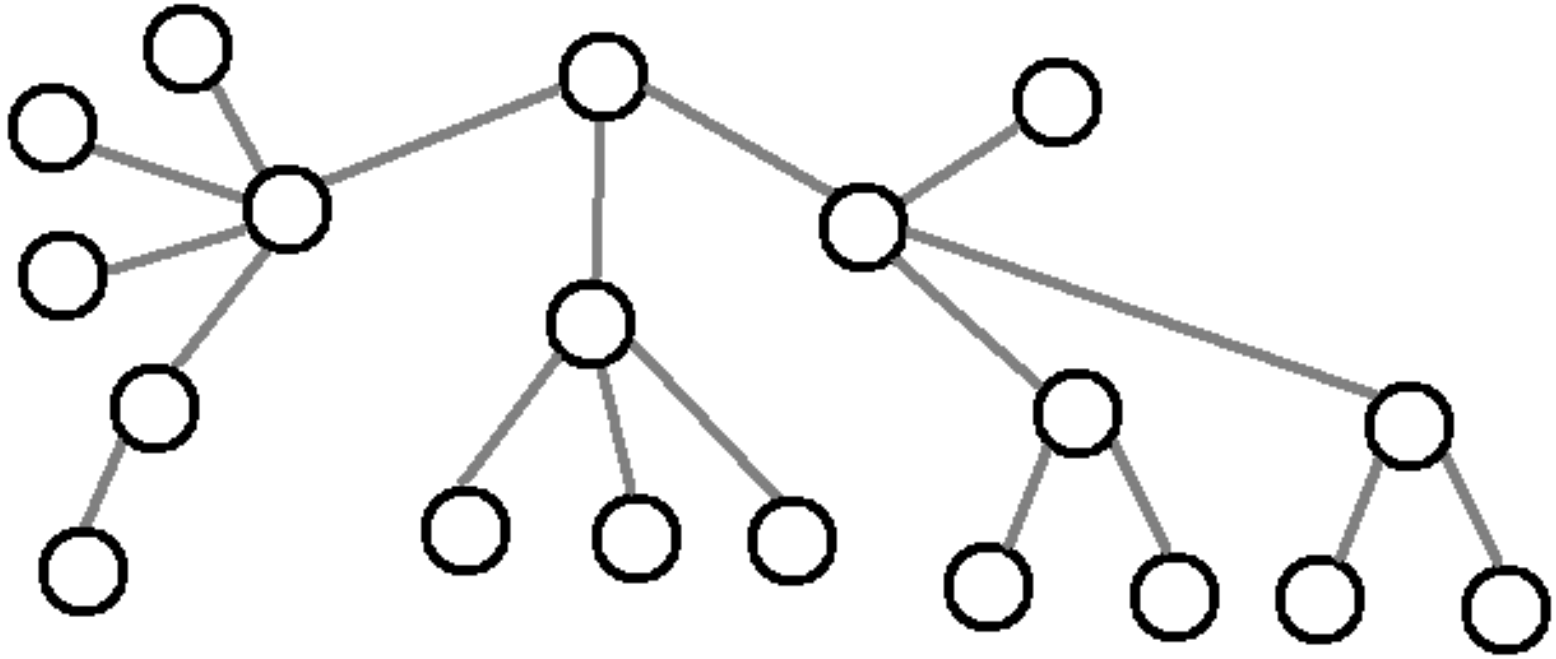
The ( ) Canonical Form for a Tree



Label each vertex with the string ( ).
While more than two vertices remain do

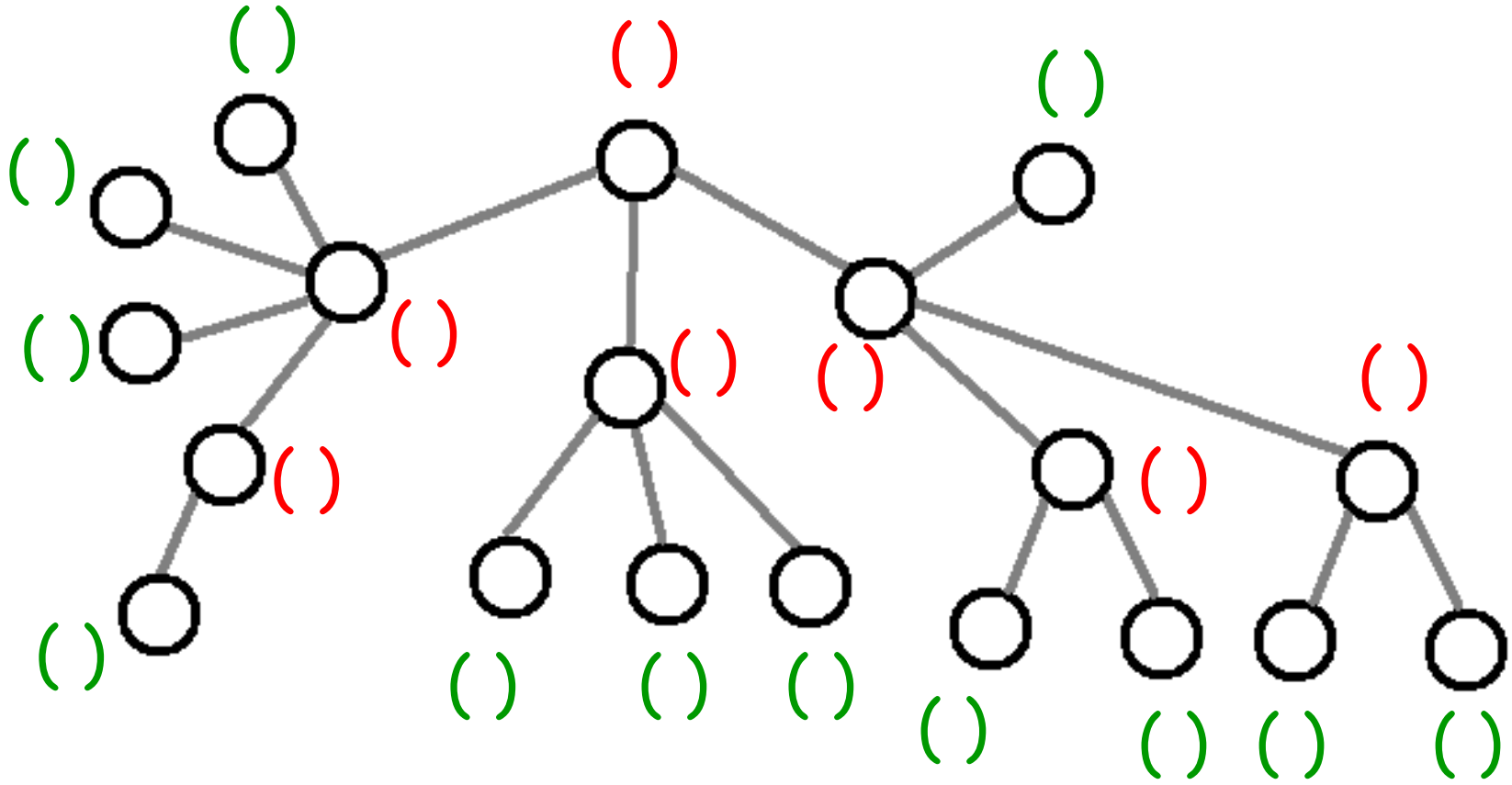
Locate all the leaves.

Remove each leaf placing its label within the (and) of the parent so that within the ( ) of the parent, the strings representing the children are sorted.

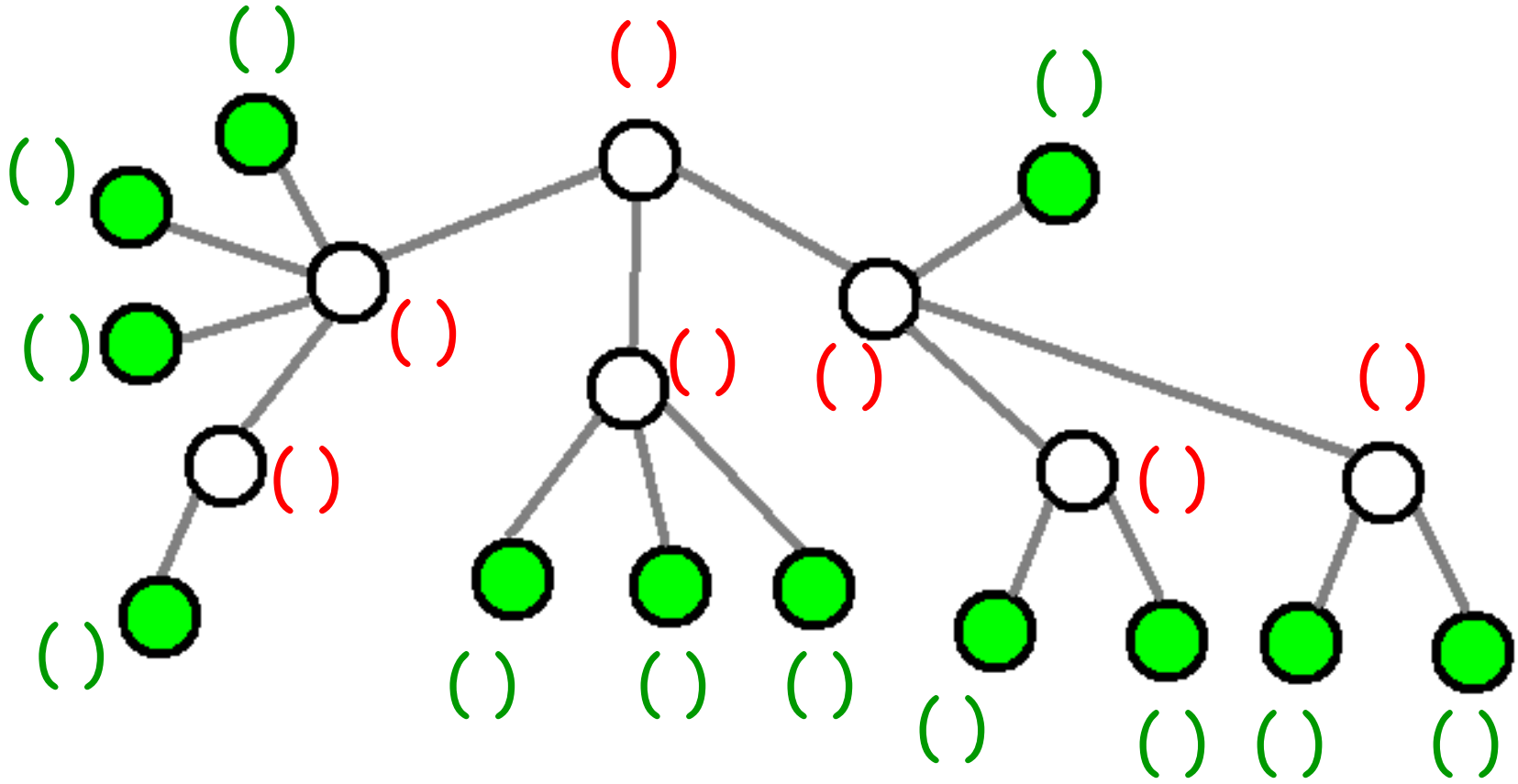
End while

If there is one vertex left, the canonical form of the tree is the label of this vertex.

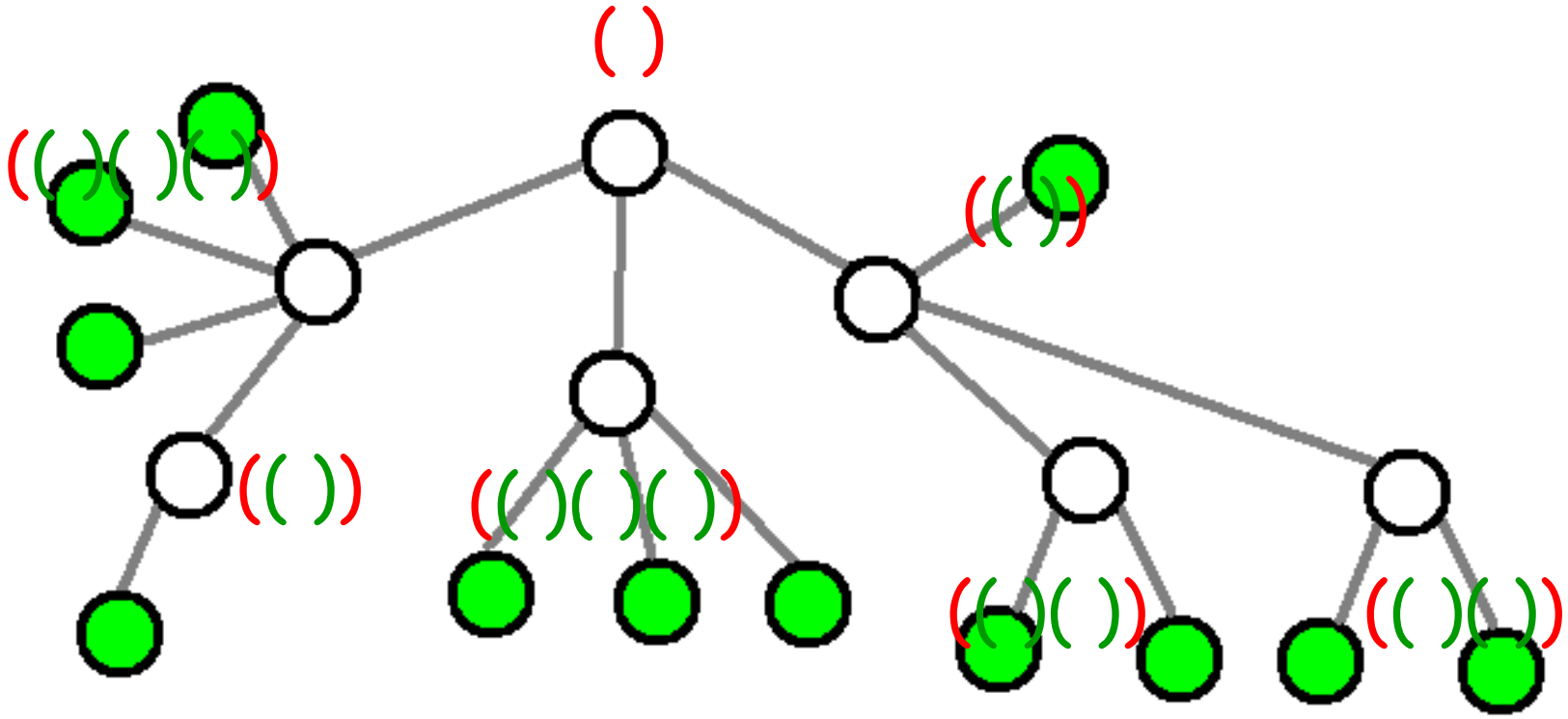
If there are two vertices u and v left, the canonical form of T is label(u) concatenated with label(v) where label(u) ≤ label(v).



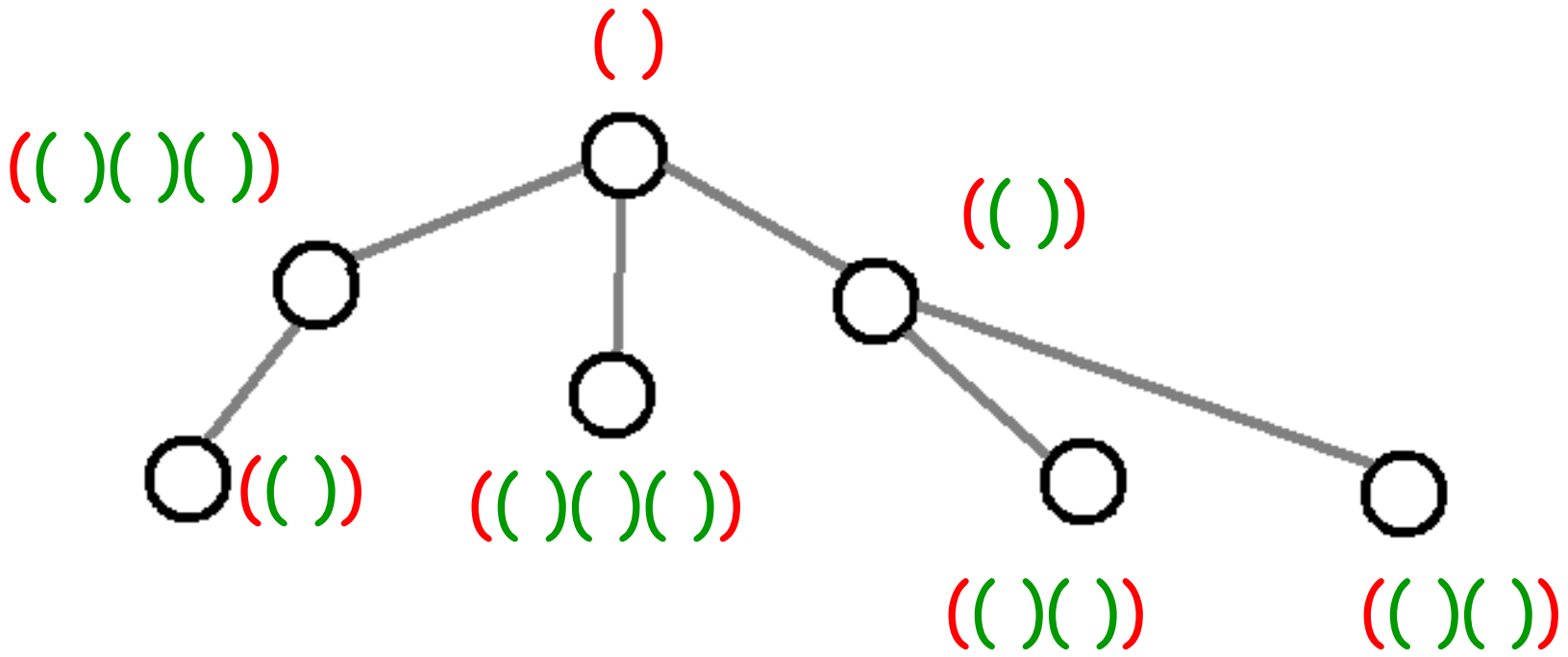
Label each vertex with ( ) .

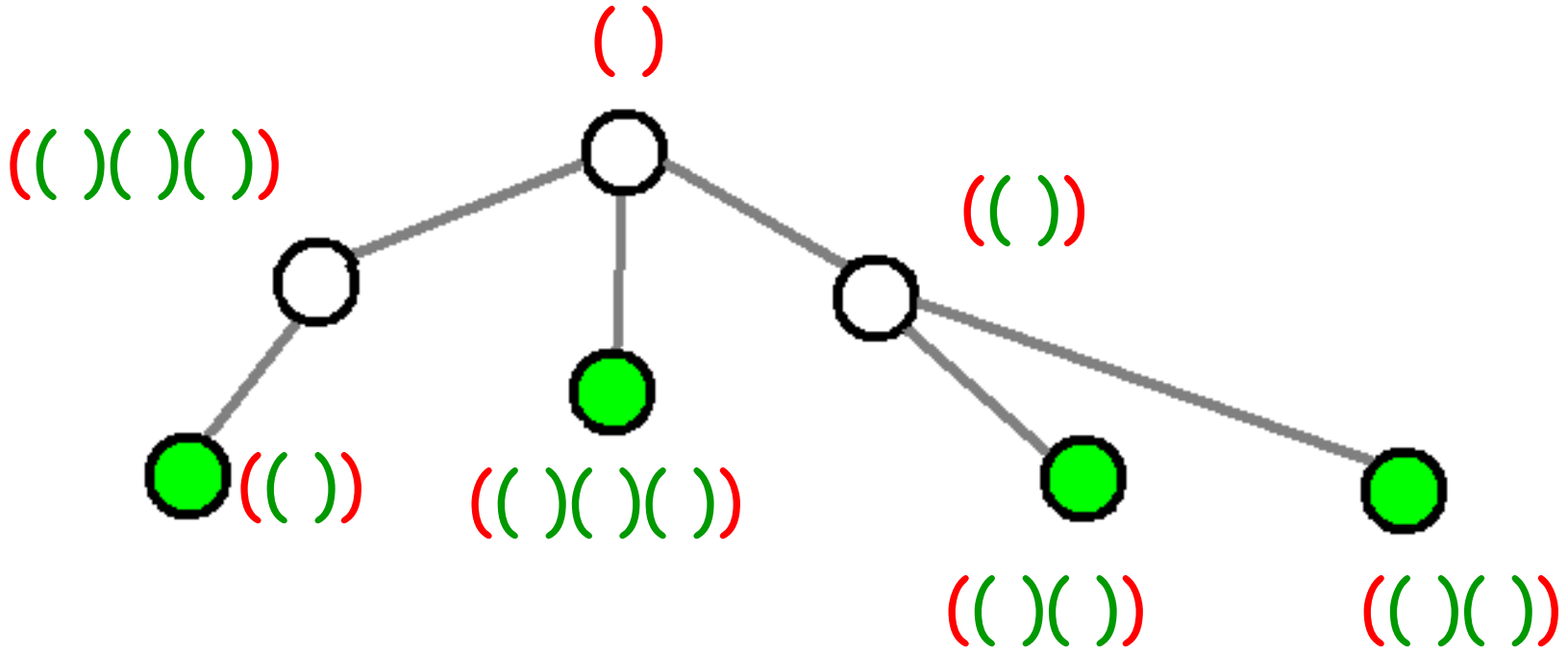


Identify the leaves.

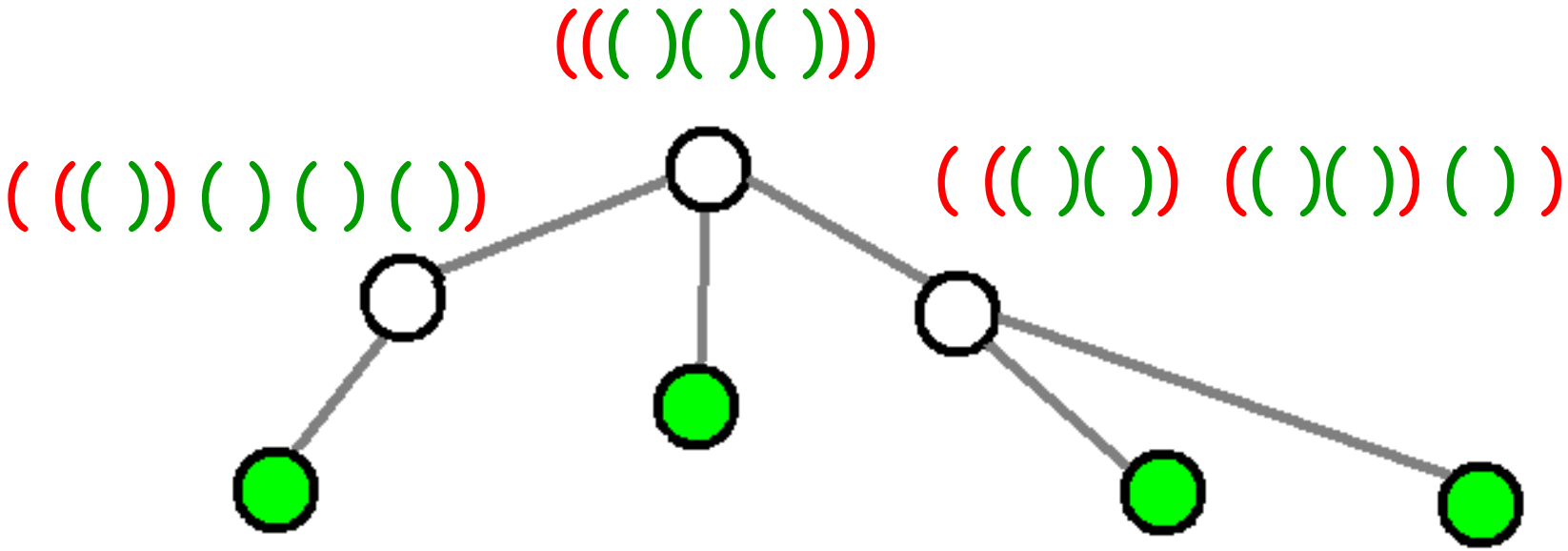


Put labels of leaves inside ( ) of their neighbours making sure strings are in sorted order.





Identify the leaves.



Put labels of leaves inside ( ) of their neighbours making sure strings are in sorted order.

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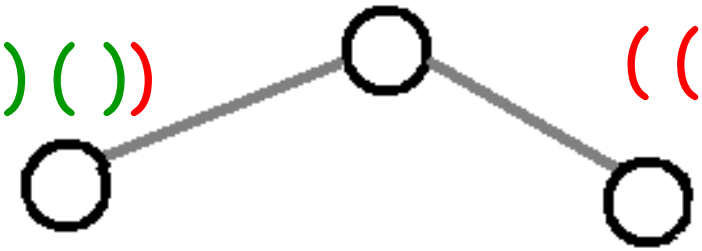

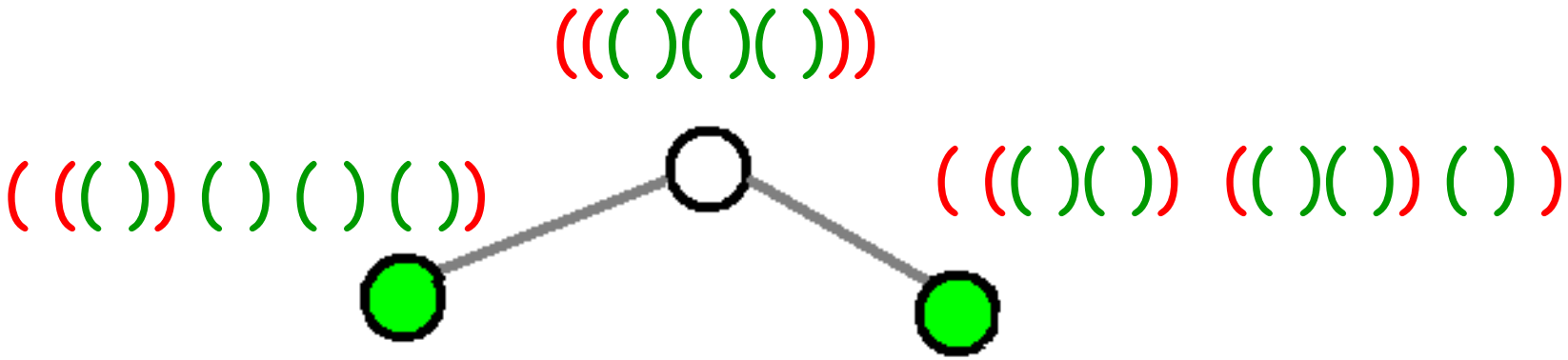

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CANONICAL FORM FOR THIS TREE:

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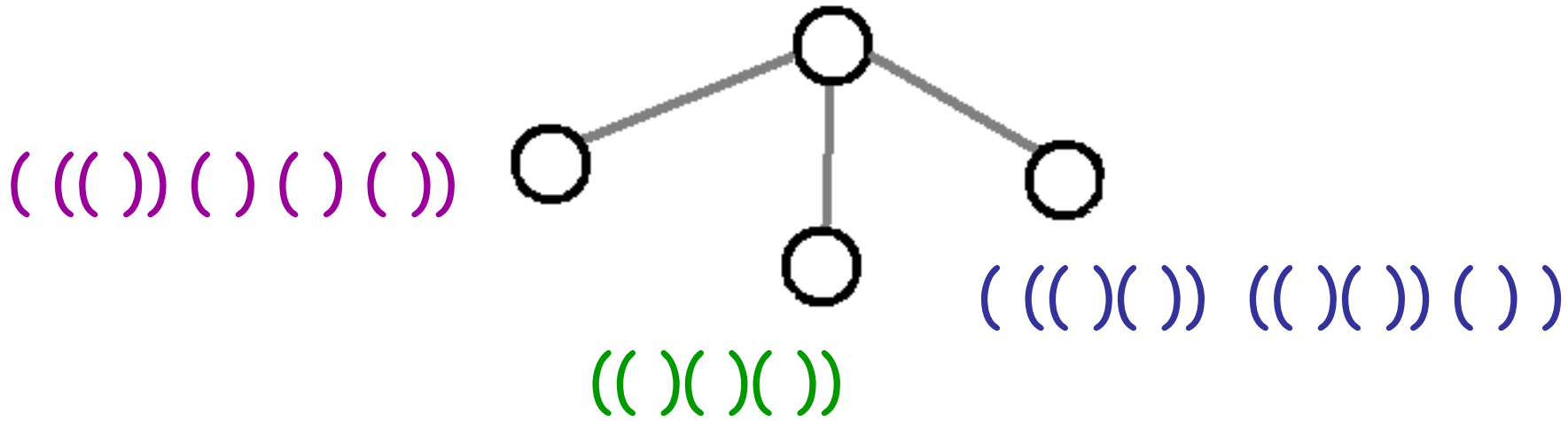
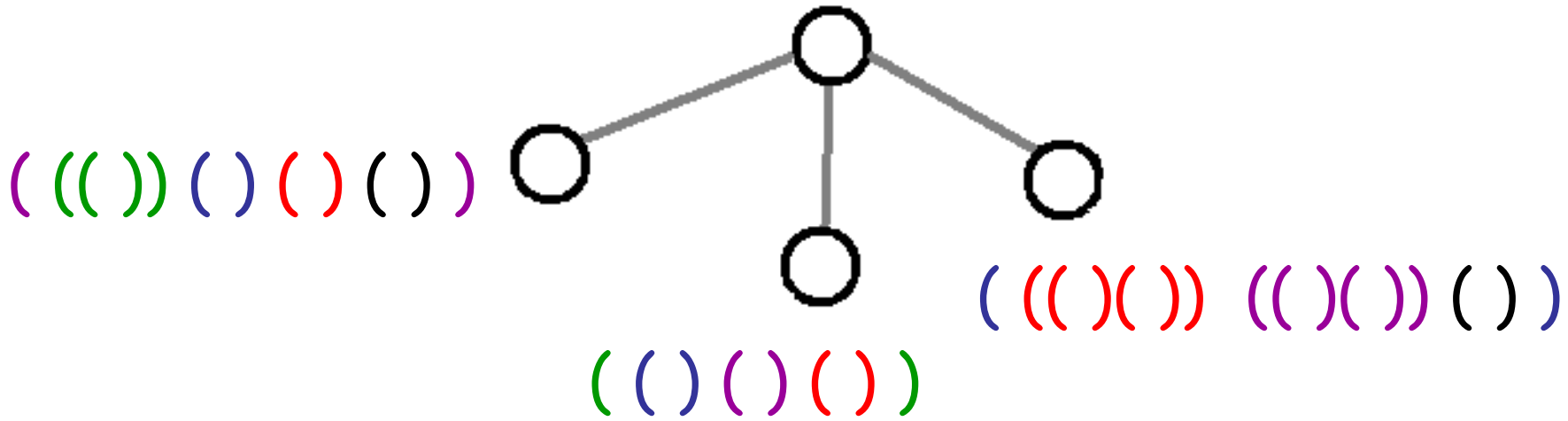
To reconstruct the tree:

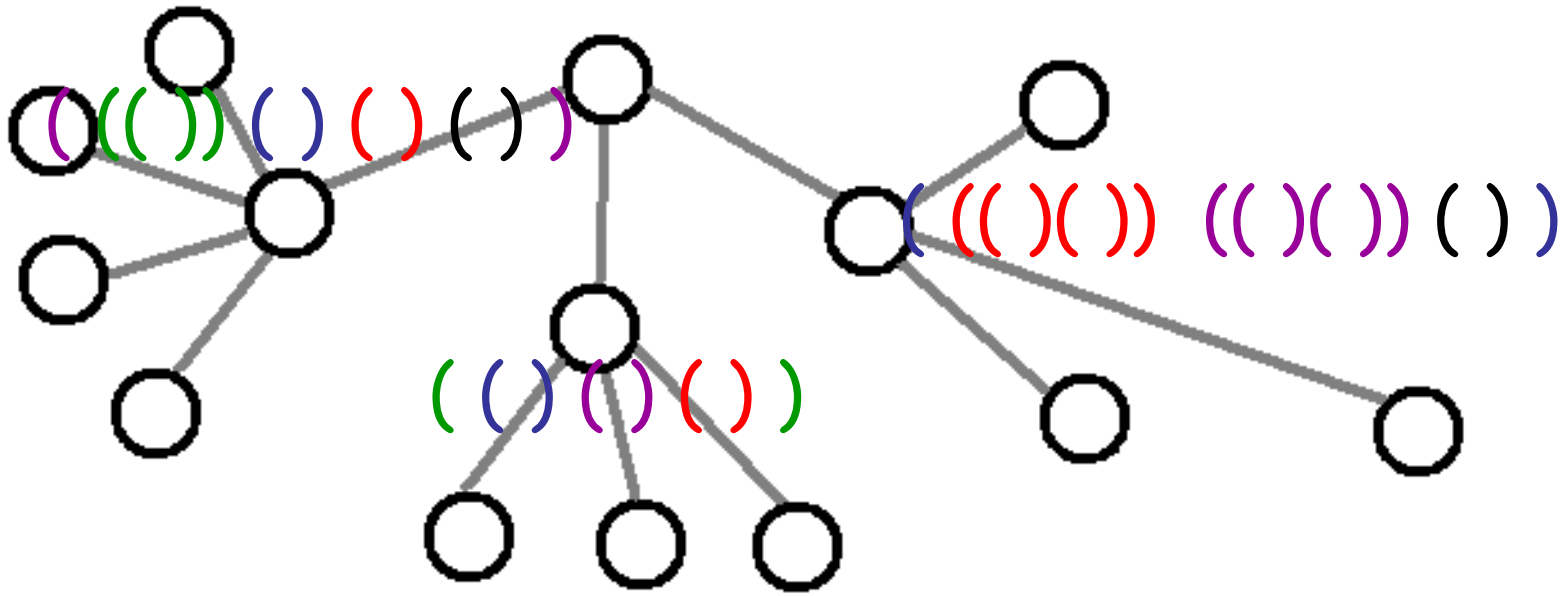
Factor what is in each ( ) into minimal well-parenthesized strings.



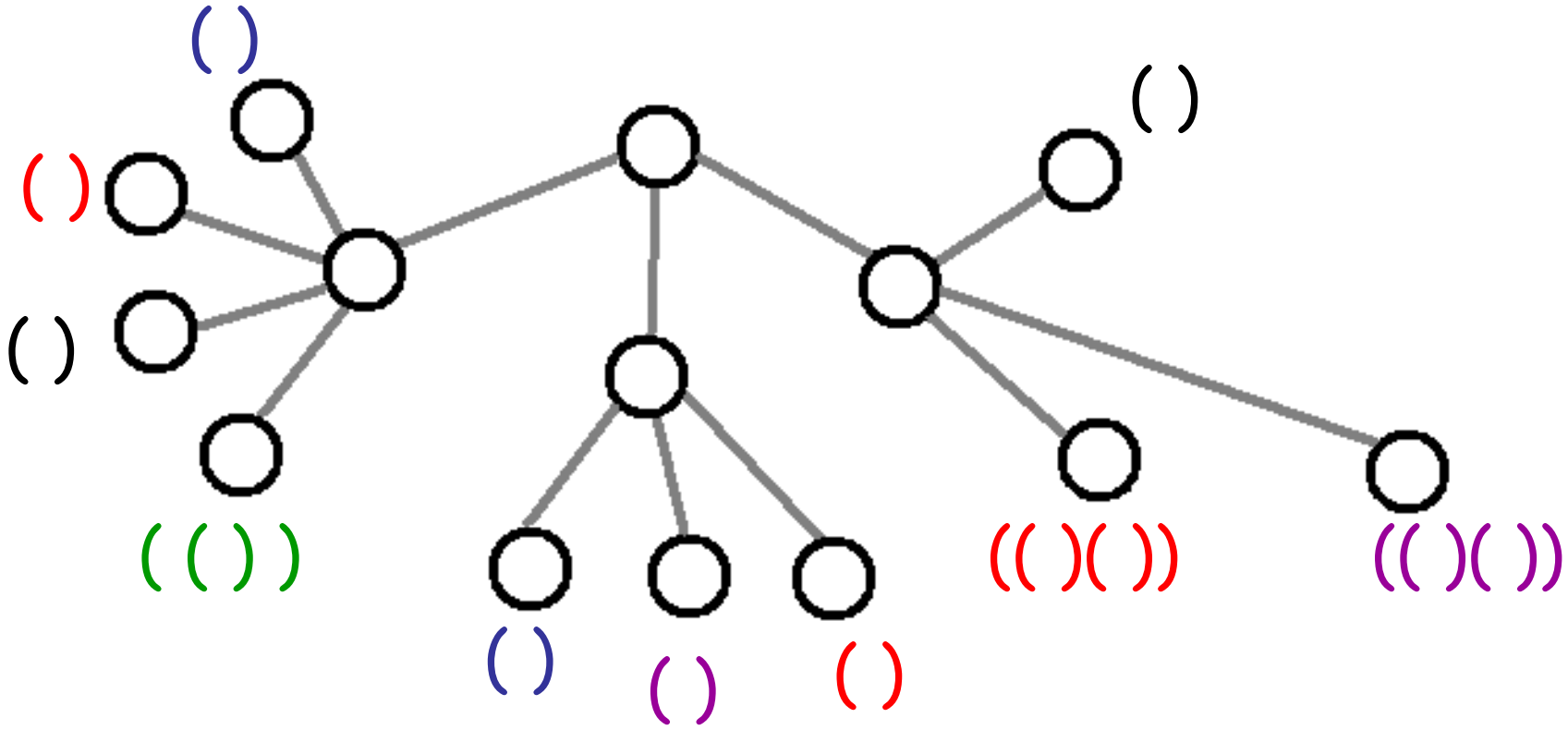
To reconstruct the tree: Add a leaf for each well-parenthesized string w with label w.



Factor what is in each ( ) into minimal well-parenthesized strings.



Add a leaf for each one.



With the labels on them.