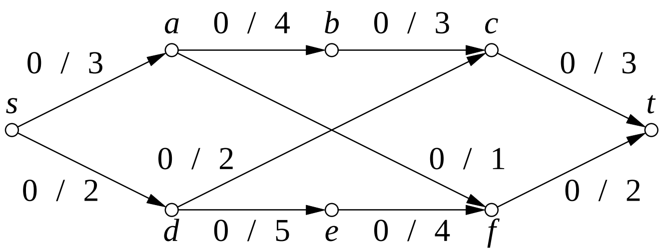
CSC 422 Summer 2017: Assignment #3

Due: Fri. July 7 at the beginning of class.

Use the attached Network Flows Worksheet to show your work for this assignment. The network N for these two questions is pictured here with an initial zero flow.



1. [30] Use the Edmonds and Karp maximum flow algorithm (described in class) to find a maximum s, t-flow in N. Visit the neighbours of each vertex in alphabetical order.

2(a) [10] Show the cut in the network N obtained by letting P be the set of vertices reachable from s in the final auxilliary graph of question #1.

(b) [10] Show that there can be more than one maximum flow in a network by giving a maximum flow for N which is different from the one you found for question #1.

3. Indicate your answer to question #3 on the attached worksheet by circling the vertices in each half of the cut.

(a) [10] For the graph G pictured on the worksheet for Question 3, find $C_1 = (P, \bar{P})$, a minimum (b, c)-cut and $C_2 = (Q, \bar{Q})$, a minimum (e, f)-cut where C_1 and C_2 are crossing cuts. Use the definition of crossing to prove that these cuts cross.

(b) [10] Use the machinery for justifying Gomery-Hu to find another minimum (e, f)-cut, $C_3 = (R, \bar{R})$, which does not cross C_1.

4. [30] Find a Gomery-Hu cut tree of the multi-graph G on the worksheet. Show your work as instructed.

Network Flows Worksheet

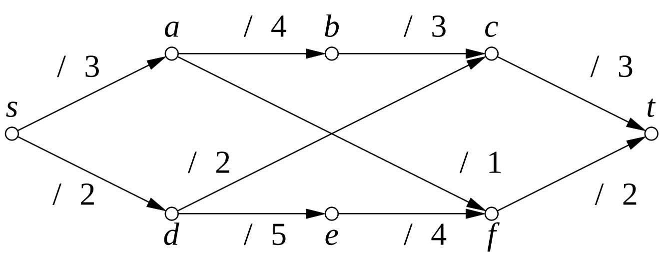
Question #1- Instructions

Use the second page to show each step of the maximum flow algorithm as follows.

1. Draw the auxillary graph on the right hand side of the page.
2. Show the augmenting path you choose from the auxillary graph by marking the edges with a red pen in the typeset graph on the LHS of the page.
3. Show the change in the flow function by filling in the new flow values for the next typeset graph on the page.

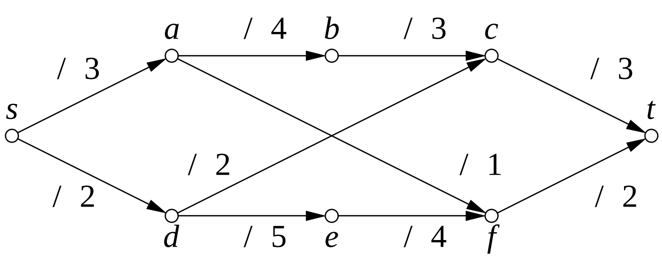
Question #2

(a) Fill in your final flow values from question #1. Indicate on the figure the vertices in P and those in $\bar{P}$.

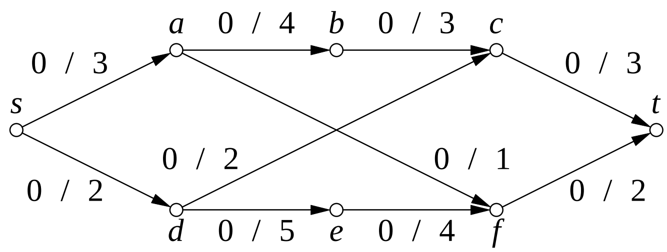


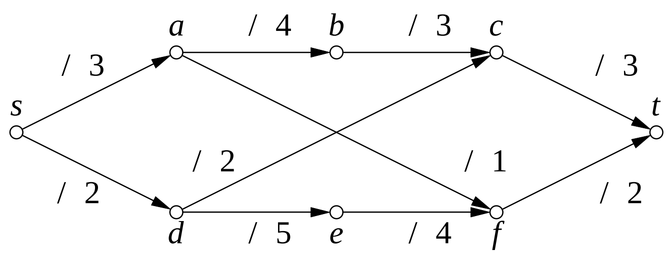
The capacity of $(P, \bar{P})$ is:

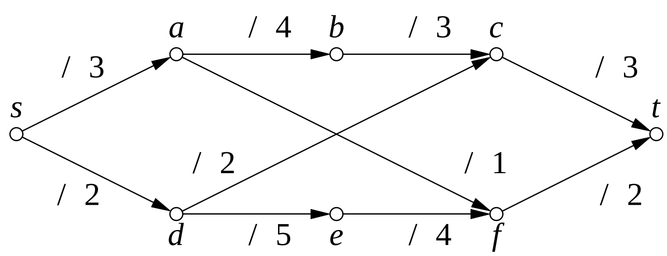
(b) Indicate an alternate maximum flow:

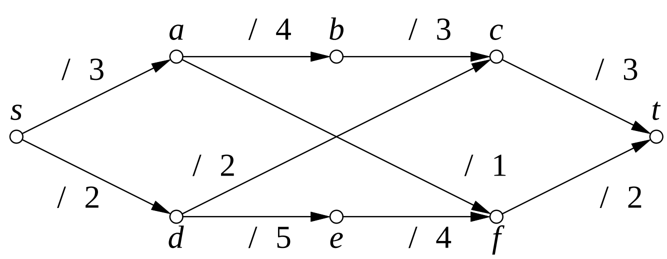


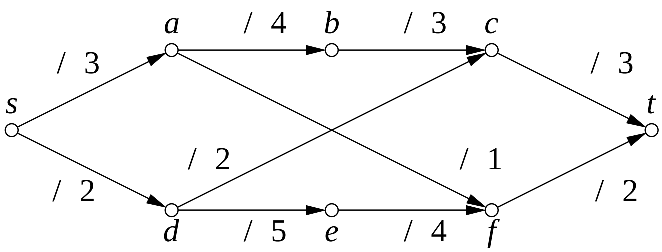
Question 1: Network Flow Worksheet



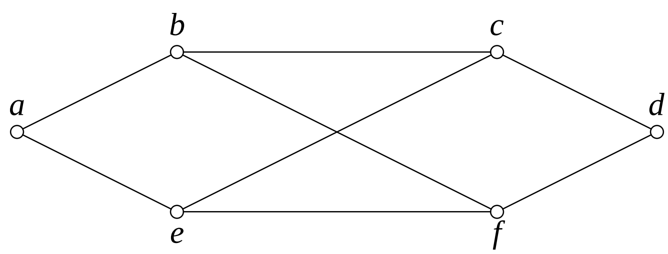




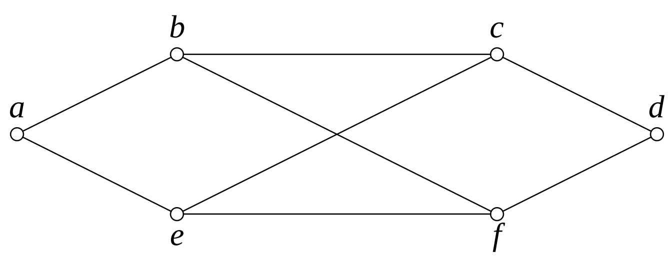




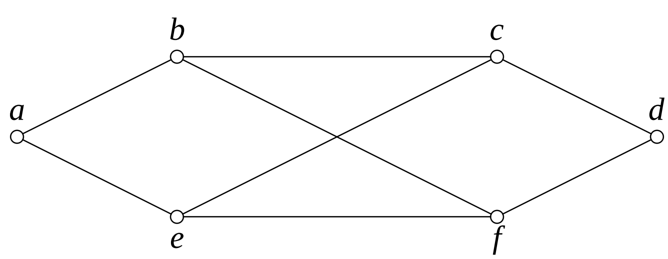
Question #3



C_1: a minimum (b, c)-cut



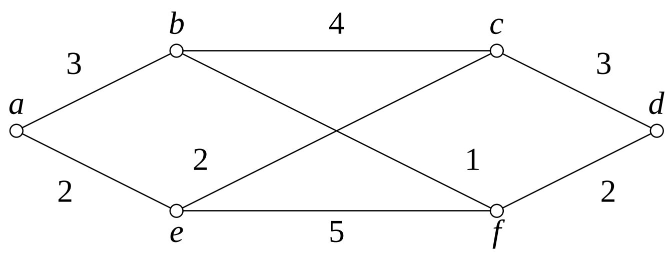
C_2: a minimum (e, f)-cut that crosses C_1.

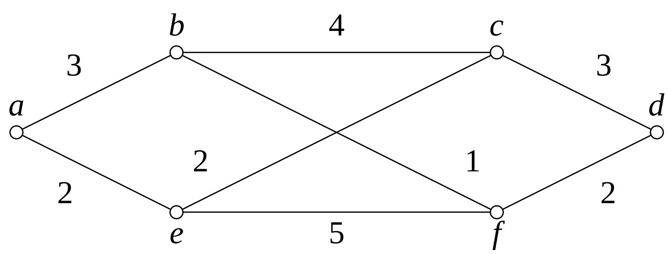


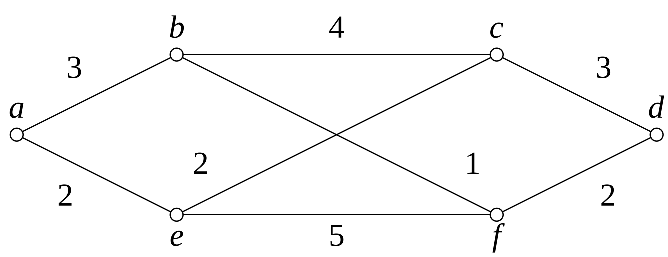
C_3: a minimum (e, f)-cut that does not cross C_1.

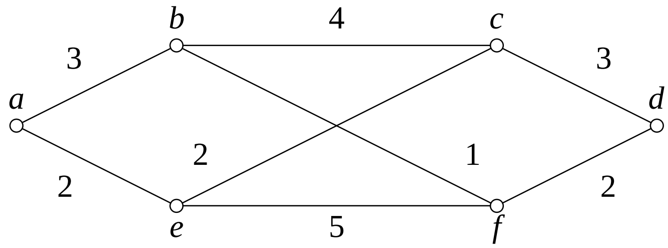
Question #4- Instructions

1. Choose the lexicographically smallest next pair of vertices at each step. Write the pair chosen beside the graph.
2. Indicate the maximum flow between this pair of vertices by filling in flow values on the figure as for question #1.
3. Indicate the minimum cut chosen by circling the vertices in each half of the cut.
4. Draw the portion of the cut tree computed so far to the right of the graph.








b
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c
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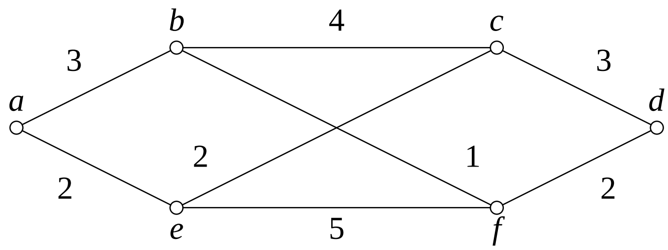
2
e
5
f


b
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