$L = \{ a^p b^q c^r d^s : p, q, r, s \geq 0, p+q = r+s \}$

1. Design a PDA that accepts L.

Hint: use four states, one for reading each different type of symbol.

2. Design a context-free grammar that generates L.

A grammar for L_2:

$L = \{ a^p b^q c^r d^s : p, q, r, s \geq 0, p+q = r+s \}$

Meaning:

S: match a with d

T: match a with c

U: match b with d

V: match b with c

$L = \{ a^p b^q c^r d^s : p, q, r, s \geq 0, p+q = r+s \}$

Meaning:

S: match a with d

T: match a with c

U: match b with d

V: match b with c

Start symbol: S

$S \rightarrow a\,S\,d$

$S \rightarrow T$

$S \rightarrow U$

$S \rightarrow V$

$T \rightarrow a\,T\,c$

$T \rightarrow V$

$U \rightarrow b\,U\,d$

$U \rightarrow V$

$V \rightarrow b\,V\,c$

$V \rightarrow \varepsilon$

Design a PDA which accepts the language

$\{ u\, u^R\, v\, v^R : u, v \in \{a,b\}^+ \}$

Hint: A bottom of the stack symbol could prove useful in helping to ensure that u matches with u^R and v matches with v^R.

Languages which are context-free.

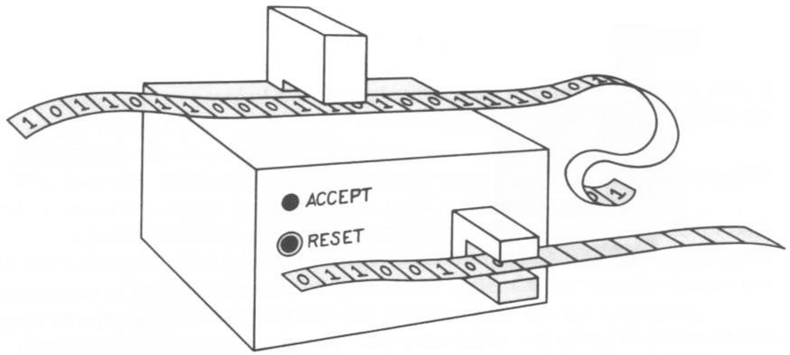
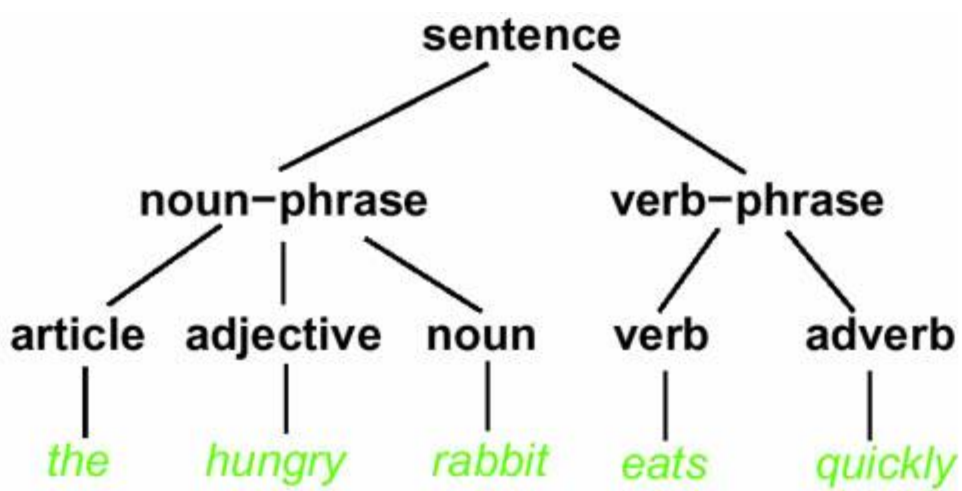
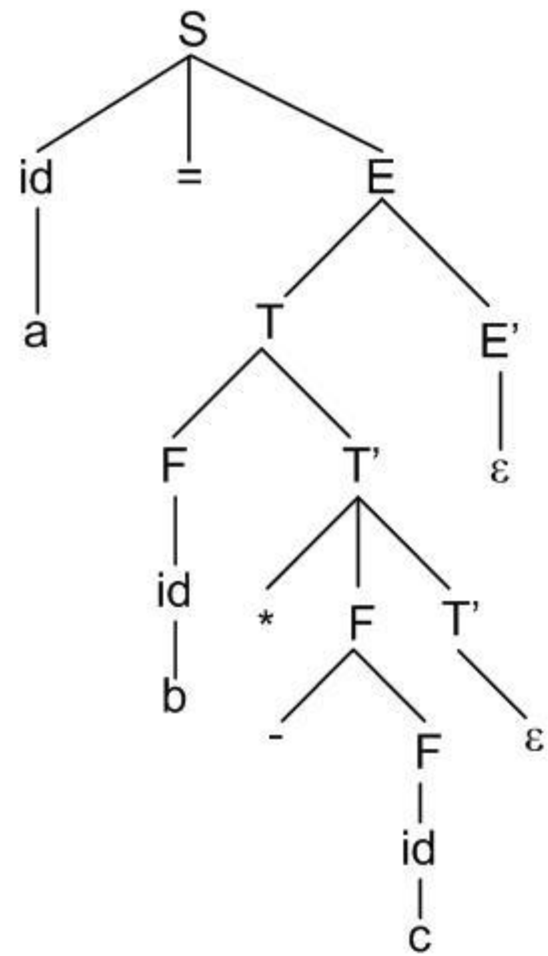


Figure 7.4 A push-down automaton



A Derivation Tree



Theorem:

If L is L(G) for some context-free grammar G, then there is a PDA M which accepts L.

Proof: By construction of a PDA which mimics derivations in the grammar.

A context-free grammar, start symbol S:

$S \rightarrow \varepsilon$

$S \rightarrow A\ B\ A$

$A \rightarrow aa$

$B \rightarrow b\ S\ a$

Apply construction to get a PDA.

Theorem: If L is a language which is accepted by some PDA M, then there is a context-free grammar which generates L.

Proof: The basic idea of the proof is to first define simple PDA's (except for at the start, one symbol is popped and 0,1, or 2 symbols are pushed for each transition).

They then show how to construct a context-free grammar from a simple PDA.

You are not responsible for this proof.