1. Create a NDFA which accepts the language generated by this context-free grammar. Start symbol: S

$S \rightarrow aa\ S$

$S \rightarrow \varepsilon$

$S \rightarrow M$

$M \rightarrow bbb$

$M \rightarrow ab\ M$

$M \rightarrow b\ S$

$M \rightarrow E$

$E \rightarrow aa$

2. List all the strings in $L = (aba \cup \Phi)(a \cup b)$

Announcements

The midterm is in class on Wed. June 21.
There is a tutorial on Tuesday June 13.
No tutorial on Tuesday June 20.
Midterm tutorial: Monday June 19, 6:30pm, ECS 123.
Bring any questions you have about assignments 1-3, old midterms or any other class material.

Assignment 3 is due on Friday at the beginning of class.

The preliminary final exam schedule has CSC 320 at 2pm on Monday August 14.

Parse Trees:

Definition of parse trees.

Parse trees are created when programs are compiled to aid in checking syntax and determining semantics of a program.

Leftmost derivations.

Example of how parse trees are used to prove the pumping theorem for context-free languages.

L= {w in {a, b}* : w has the same number of a's and b's}

Start symbol S

$S \rightarrow e$	$A \rightarrow a$	$B \rightarrow b$
$S \rightarrow a\ B$	$A \rightarrow a\ S$	$B \rightarrow b\ S$
$S \rightarrow b\ A$	$A \rightarrow b\ A\ A$	$B \rightarrow a\ B\ B$

S- generates strings with #a's = #b's

A- means need one "a"

B- means need one "b"

Three derivations for aabbab:

1. S ⇒ aB ⇒ aaBB ⇒ aabSB ⇒ aabbAB ⇒ aabbaB ⇒ aabbab

2. S ⇒ aB ⇒ aaBB ⇒ aabB ⇒ aabbS ⇒ aabbaB⇒ aabbab

3. S ⇒ aB ⇒ aaBB ⇒ aaBbS ⇒ aaBbaB ⇒ aaBbab ⇒ aabbab

Leftmost derivation: leftmost non-terminal replaced at each step.

Which ones are leftmost?

Two of the derivations are essentially the same- they correspond to the same leftmost derivation. The other one is different.

A context-free grammar is ambiguous if there is a string in the language which has two or more distinct parse trees (or equivalently, two or more distinct leftmost derivations).

Parse Trees- $G = (V, \Sigma, R\ S)$

Nodes of parse trees are each labelled with one symbol from $V \cup \{\varepsilon\}$.

Node at top- root, labelled with start symbol S.

Leaves- nodes at the bottom.

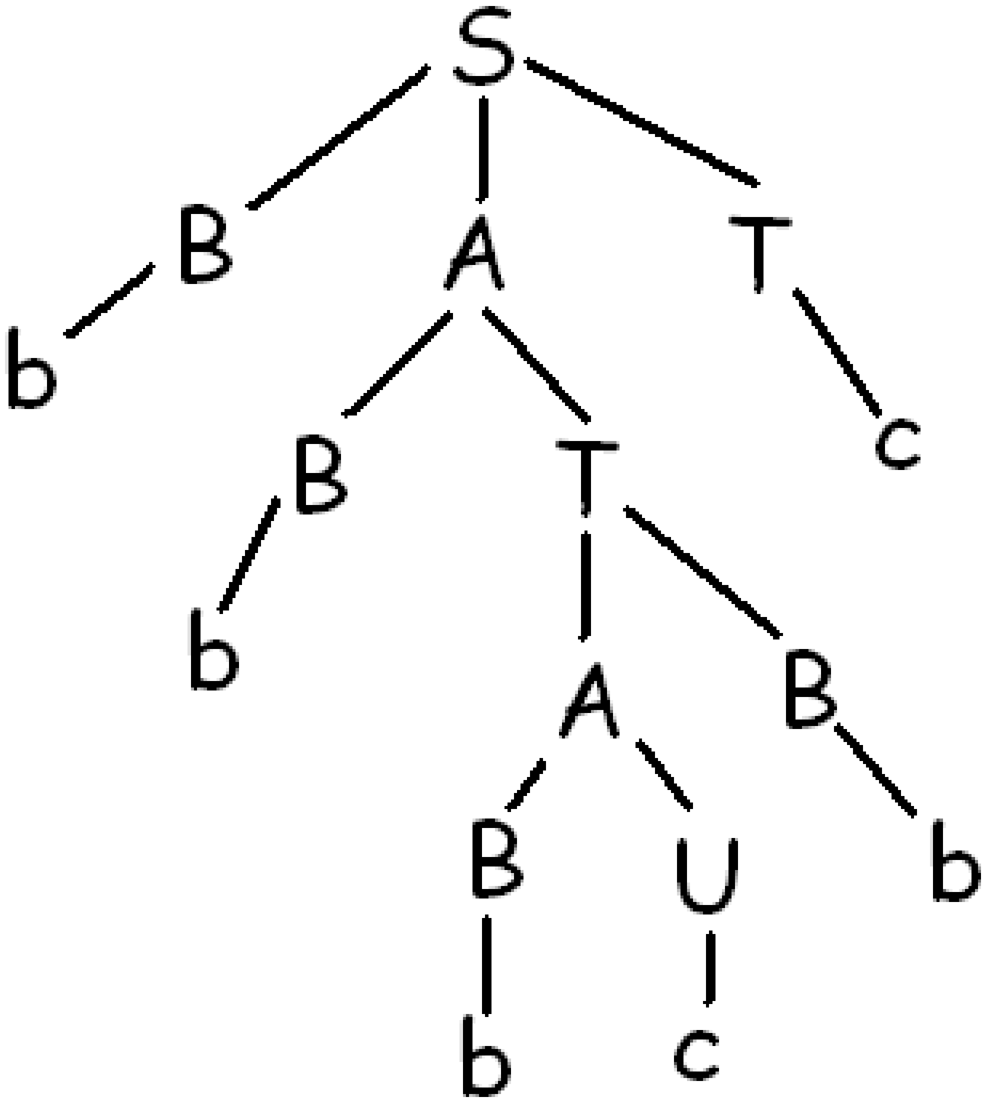
Yield- concatenate together the labels on the leaves going from left to right.

Leftmost derivation: always replace the leftmost non-terminal at each step.

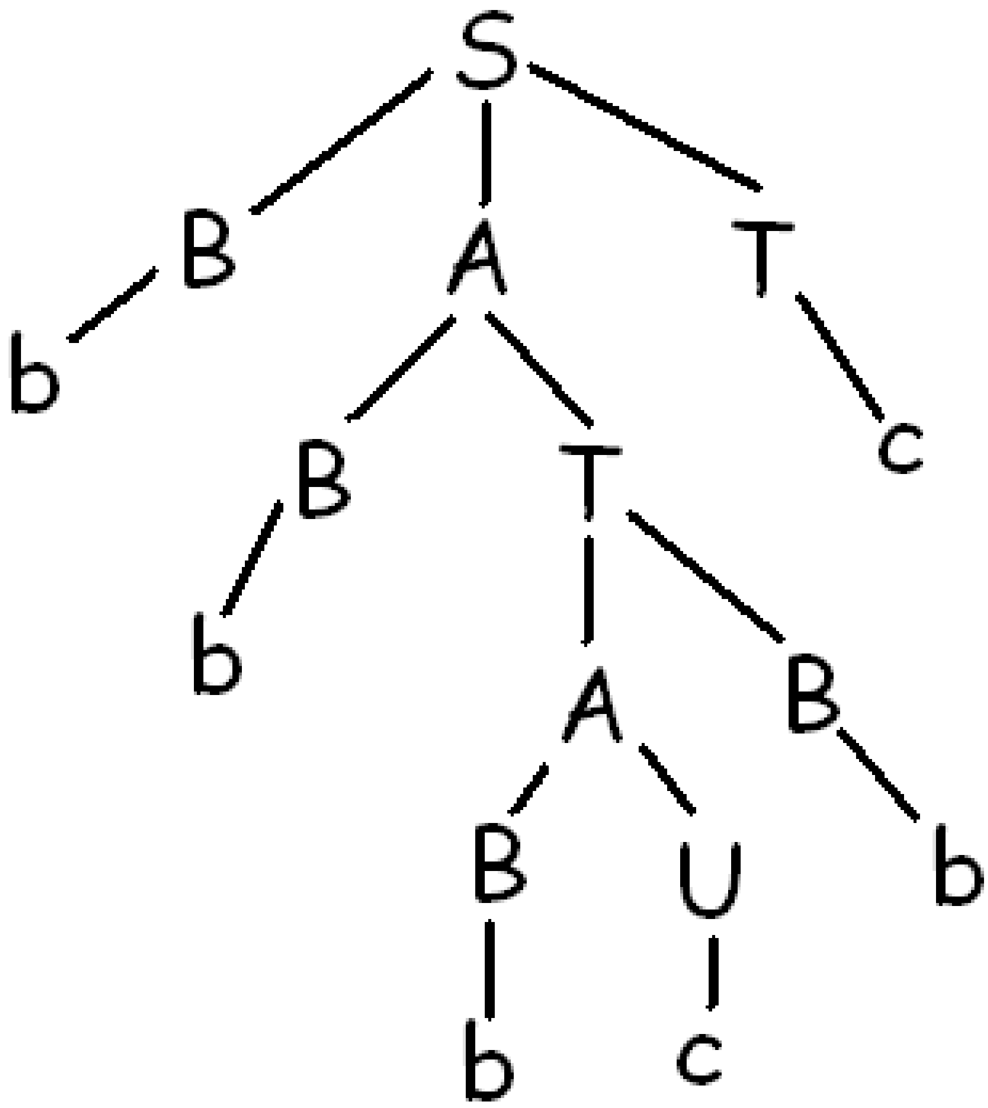
Theorem: Every derivation in a grammar G corresponds to a unique leftmost derivation.

Proof:

Construct the parse tree and determine the leftmost derivation from it.

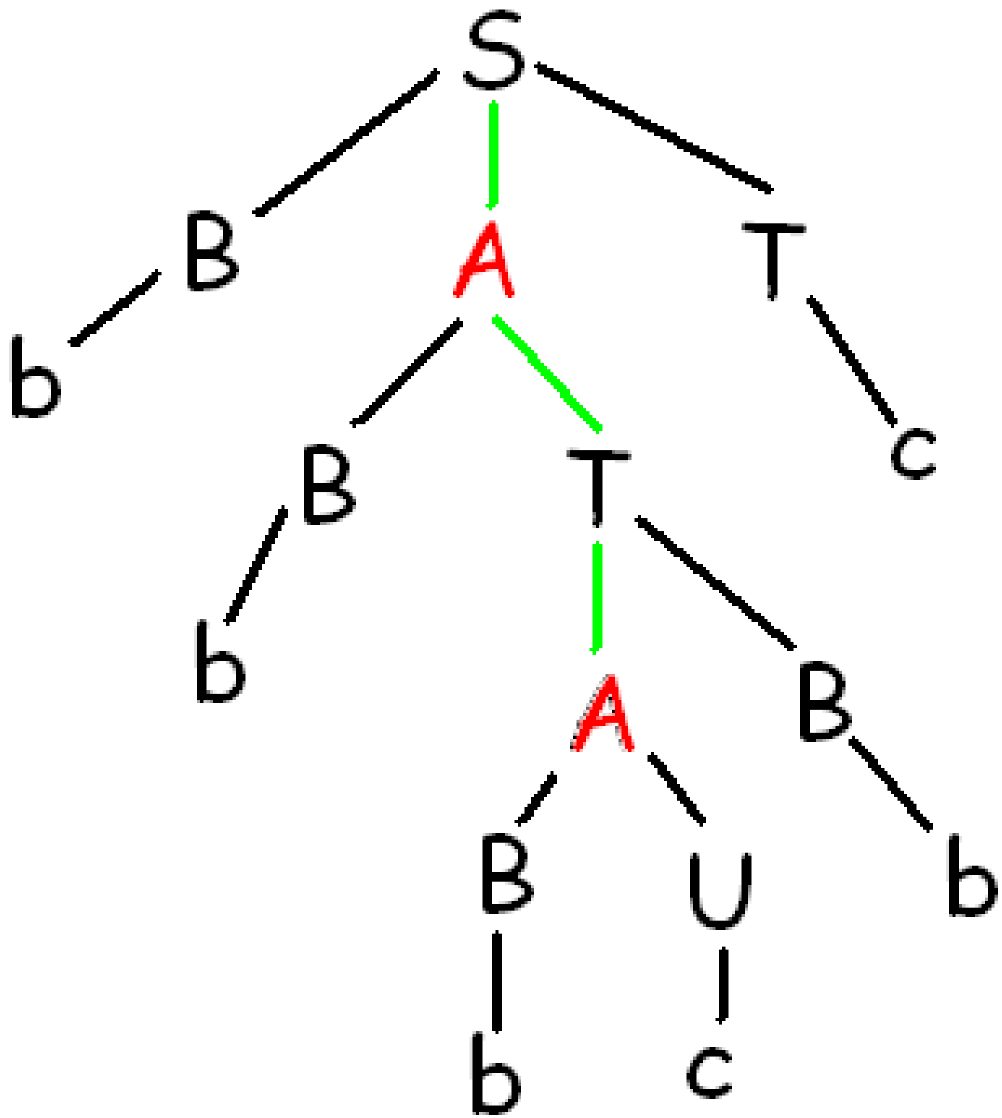


Find the leftmost derivation for this parse tree.



To pump:

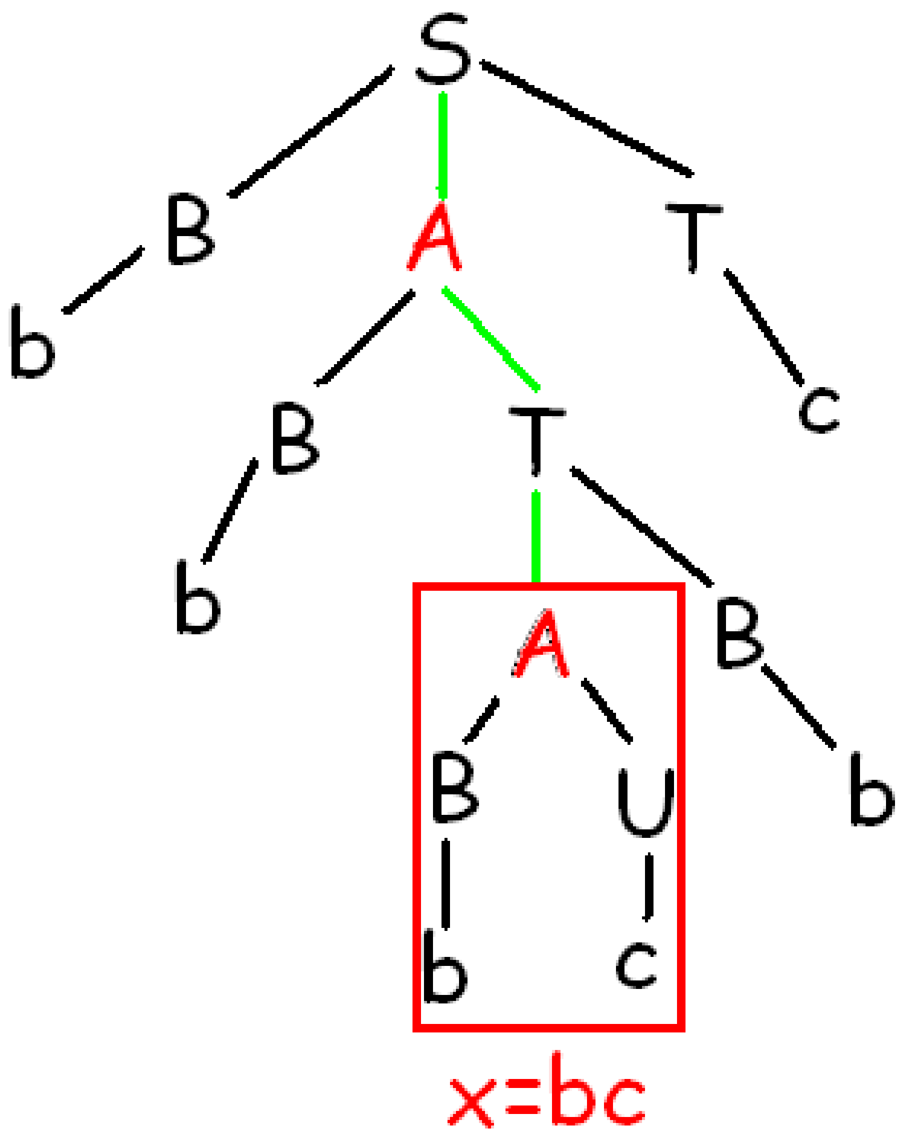
Look for a path from root with a repeated non-terminal.



Path:

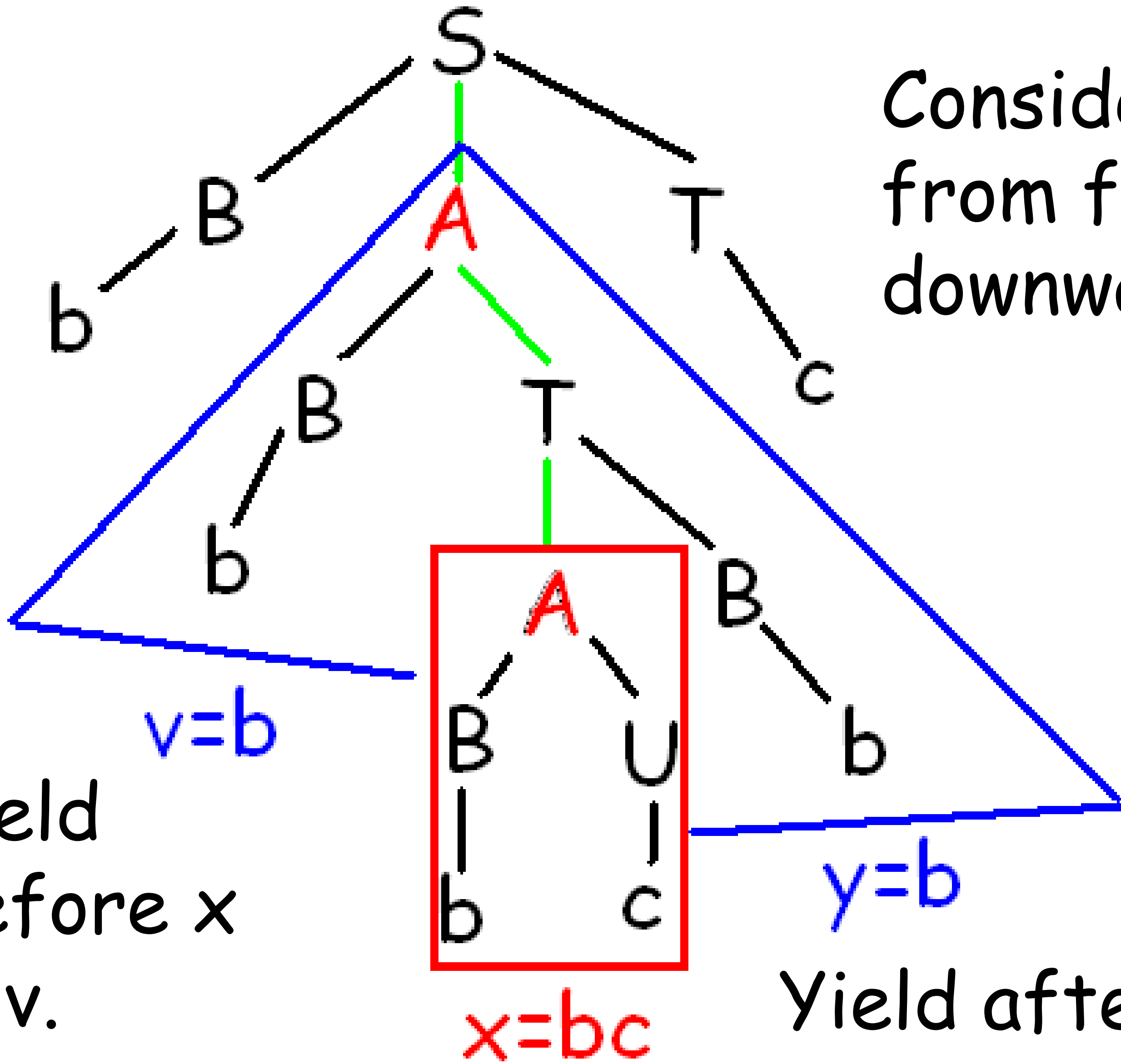
S - A - T - A

Non-terminal A is repeated.



Yield from second copy downwards gives x.

Consider tree from first copy downwards.

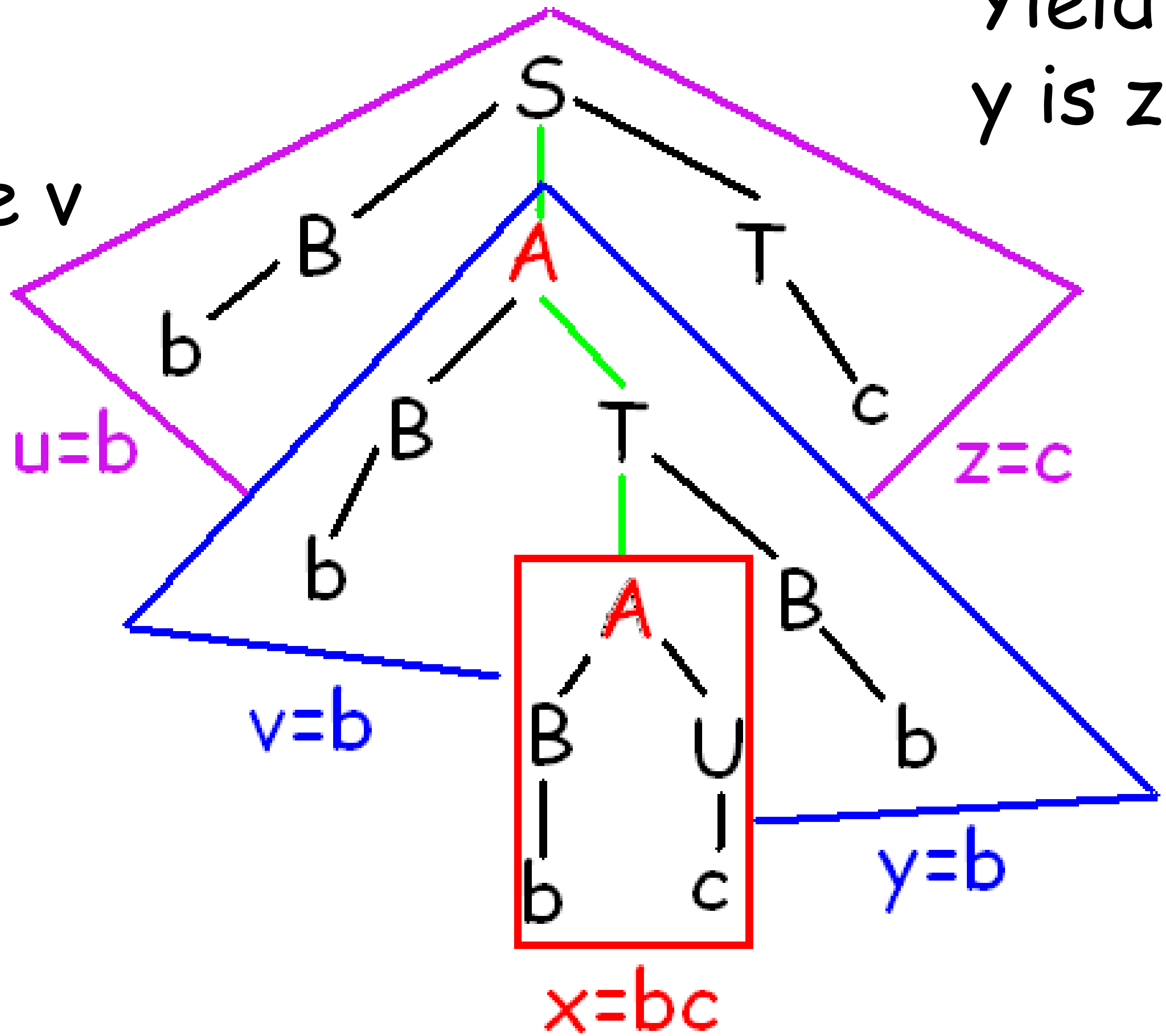


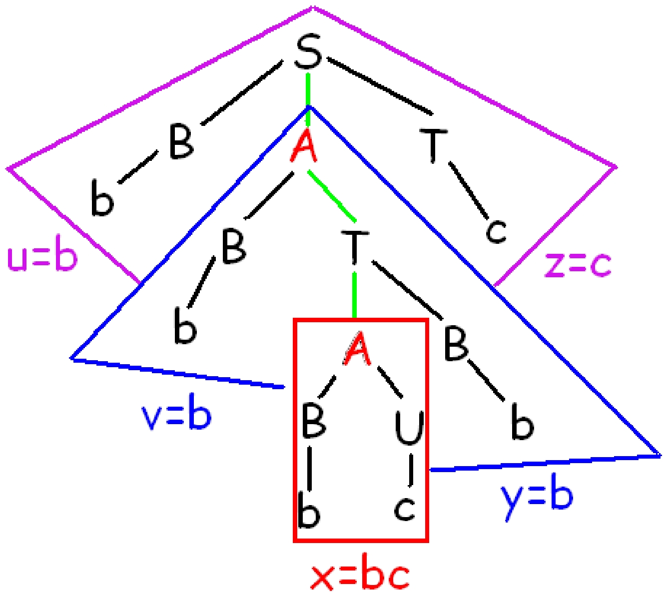
Yield before x is v.

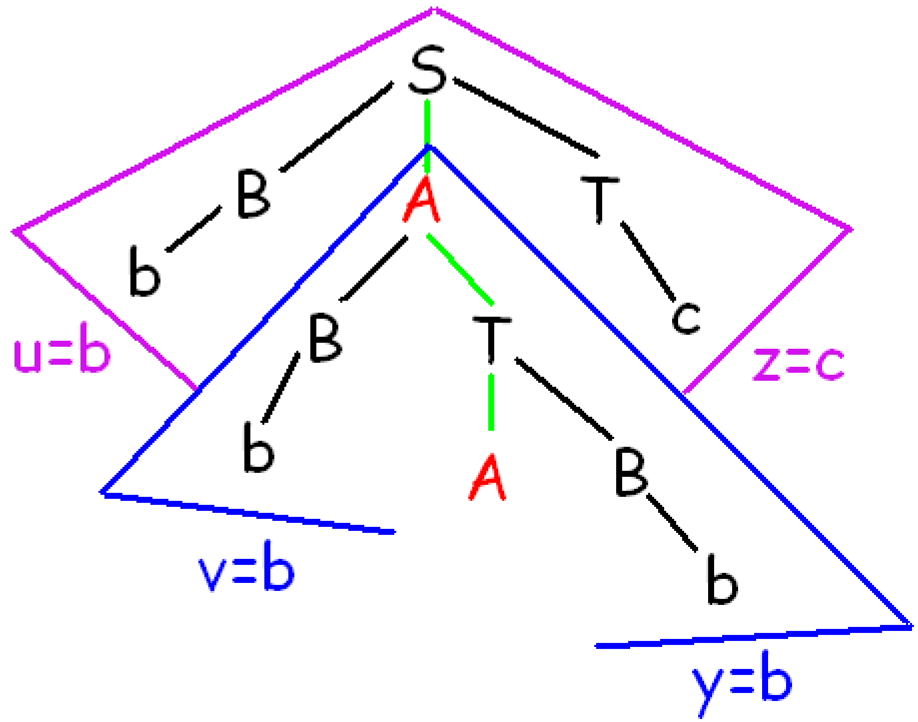
Yield after x is y.

Yield of whole tree before v is u.

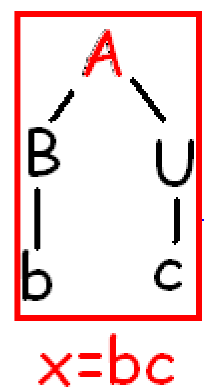
Yield after y is z.

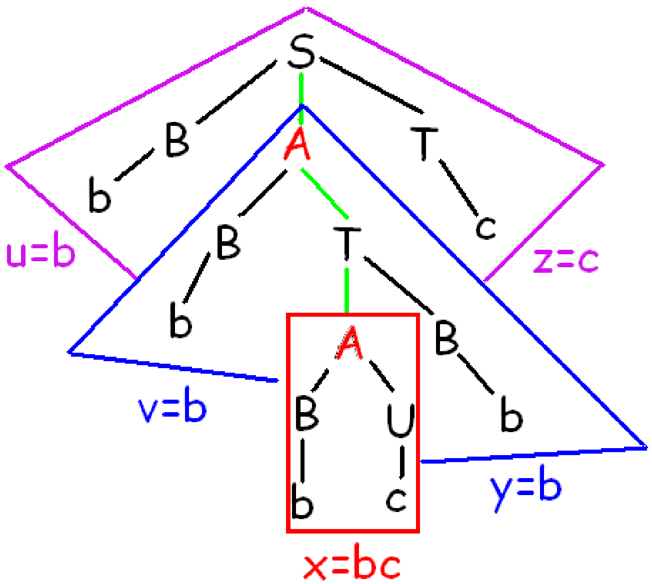




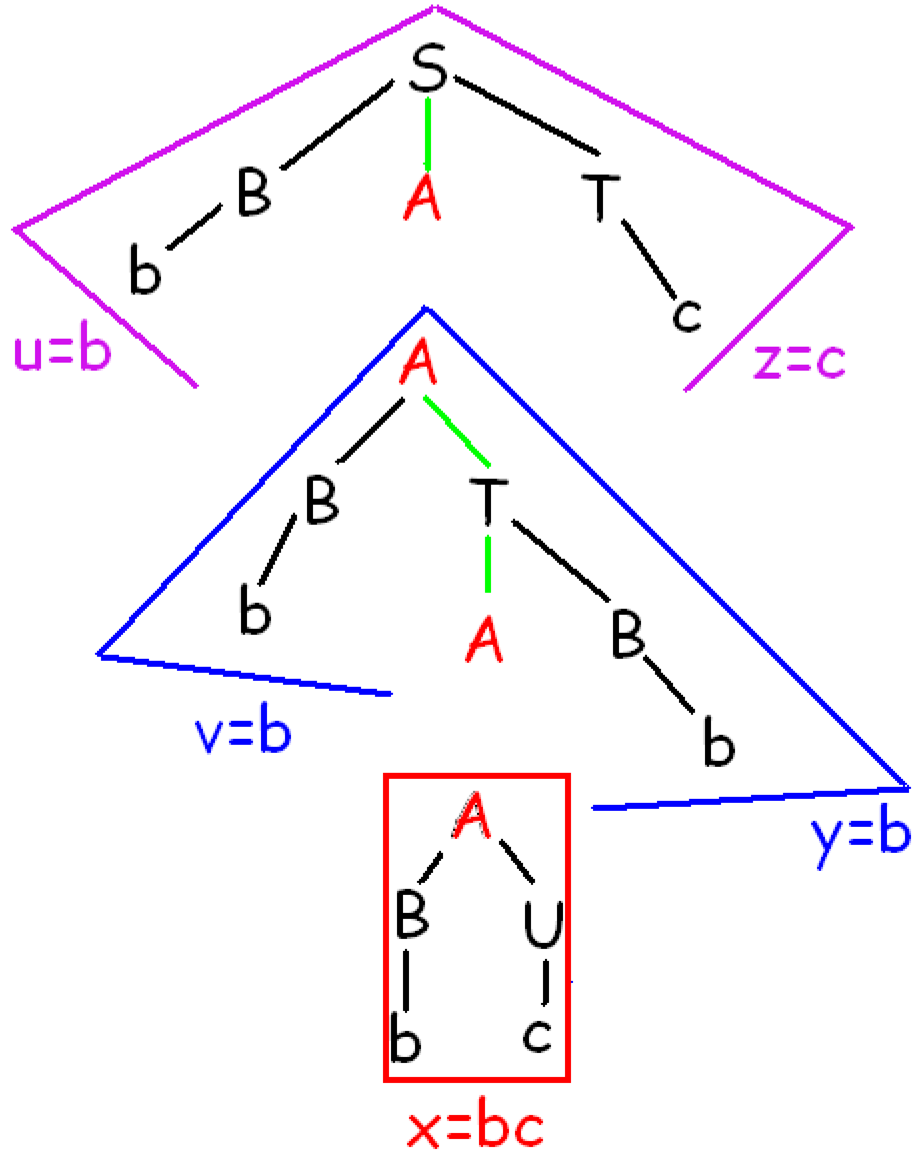


Cut off part of tree from second copy downwards.

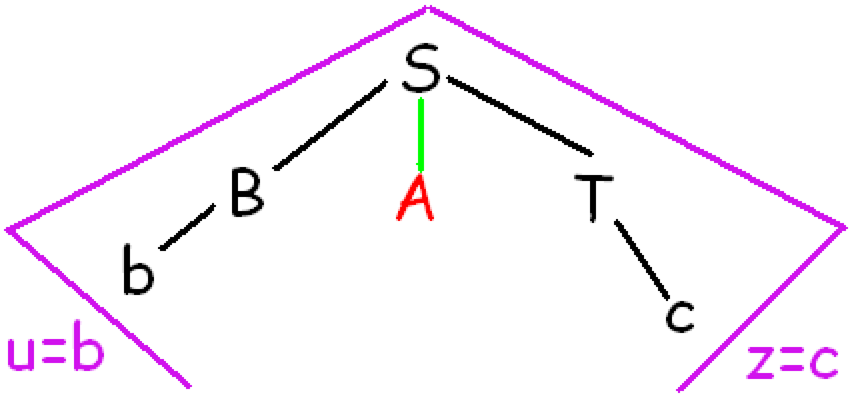


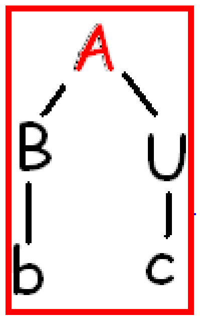


Cut off part from first copy downwards.



To pump zero times:



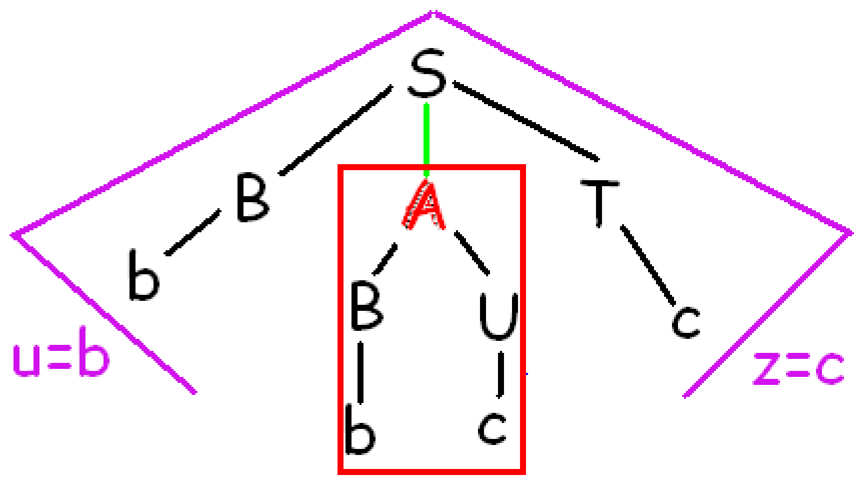


x=bc

New yield:

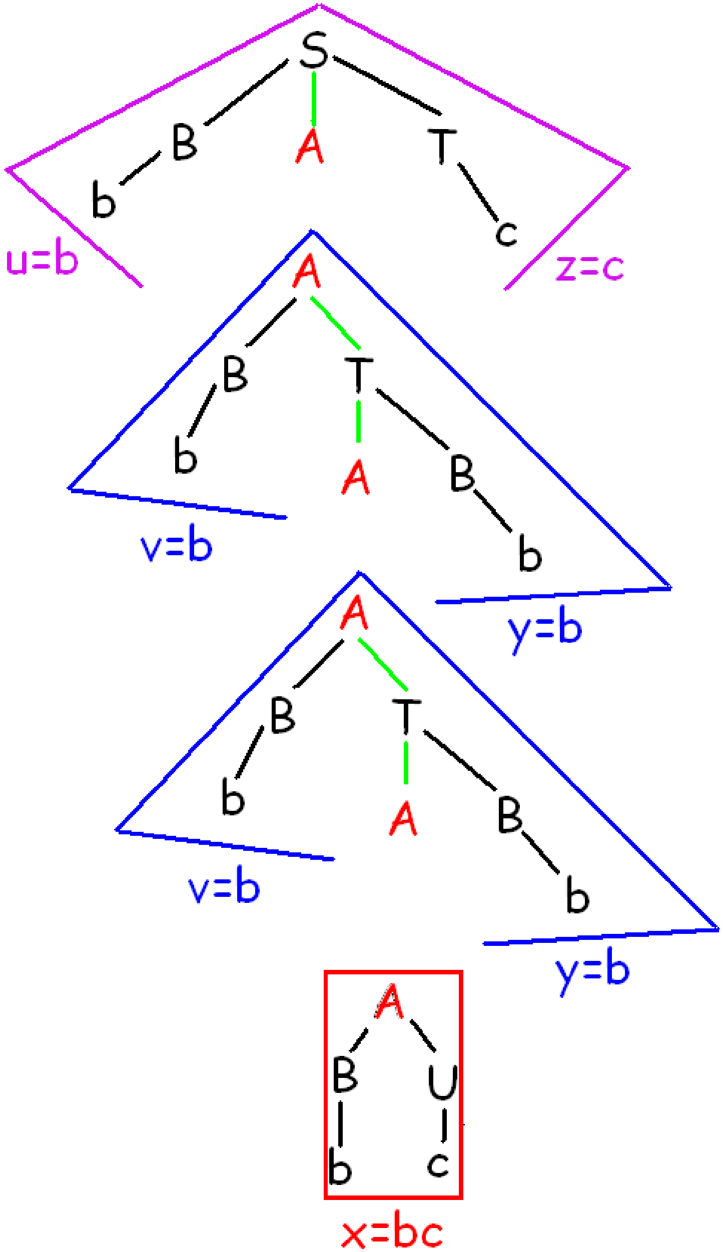
$u\ x\ z = b\ bc\ c$

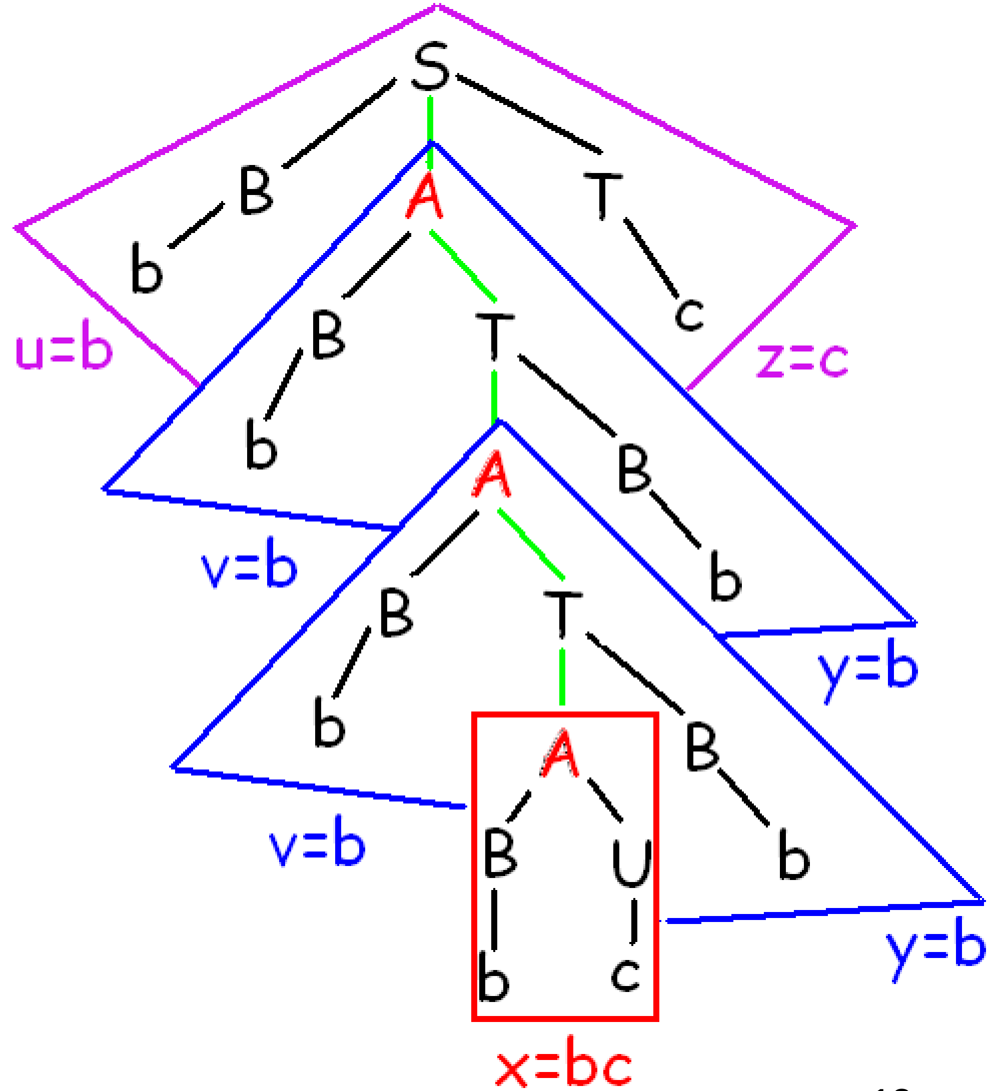
$= u\ v^0\ x\ y^0\ z$



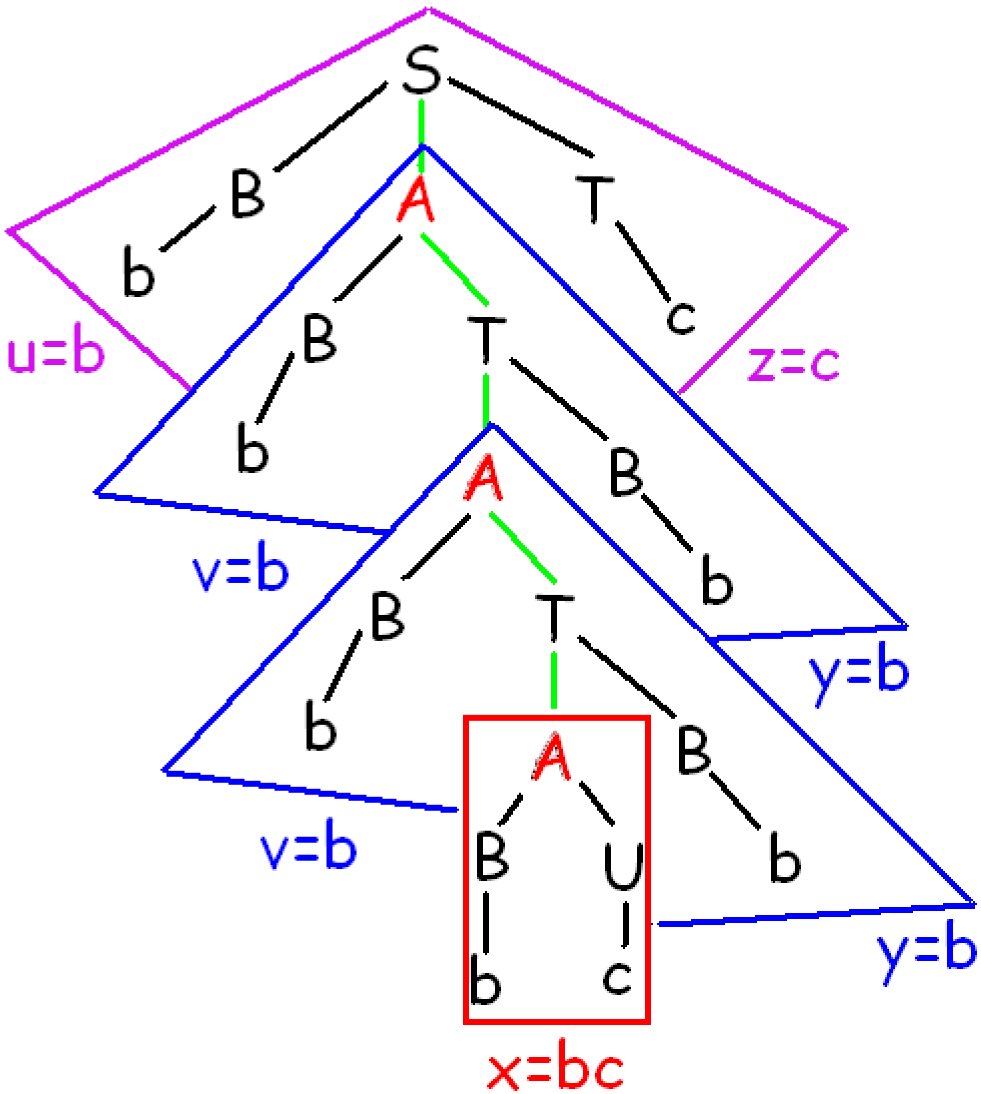
x=bc

To pump twice:





Pumping twice:



New yield is:

$u\ v\ v\ x\ y\ y\ z$

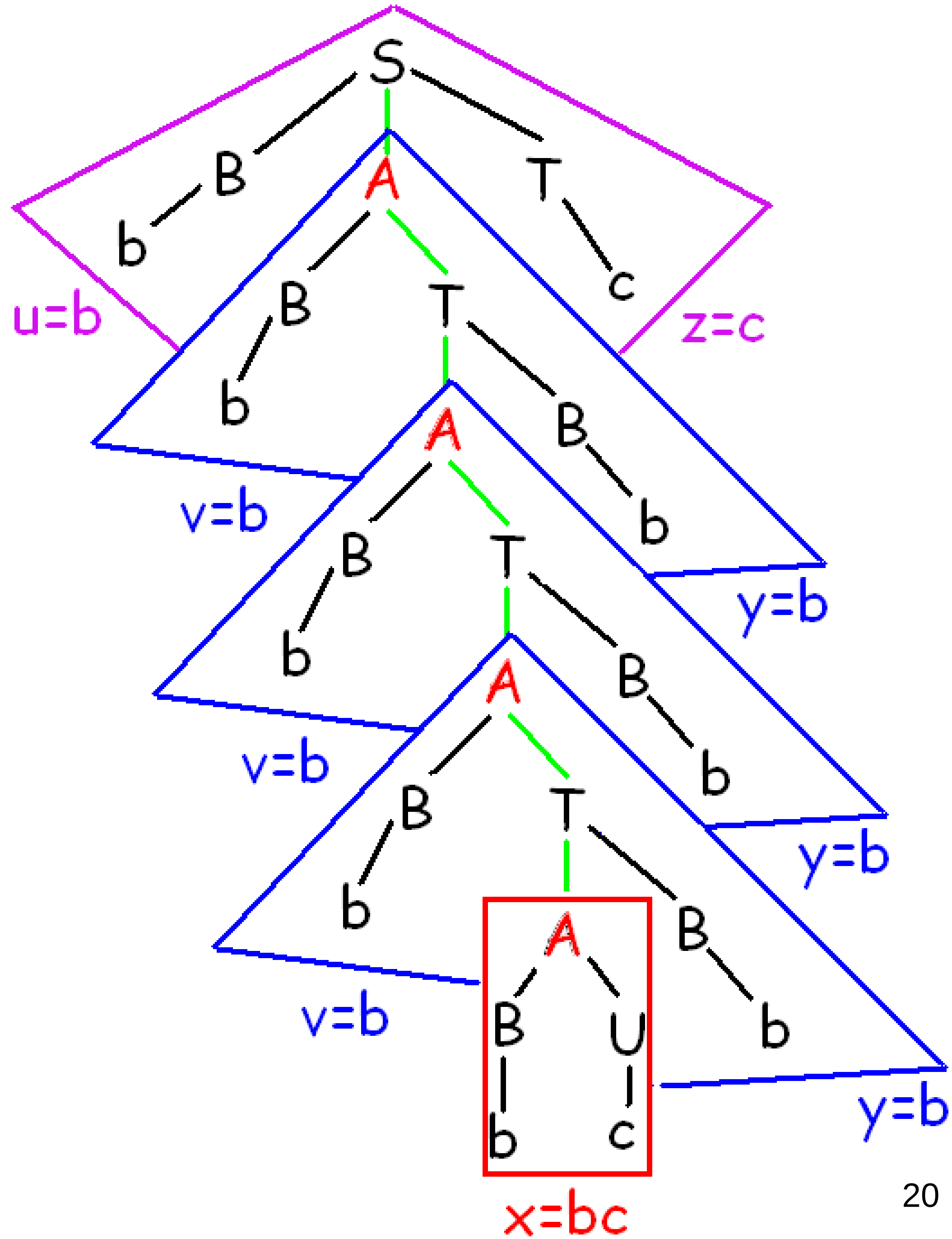
$= u\ v^2\ x\ y^2\ z$

To pump three times:

New yield is:

u vvv x yyy z

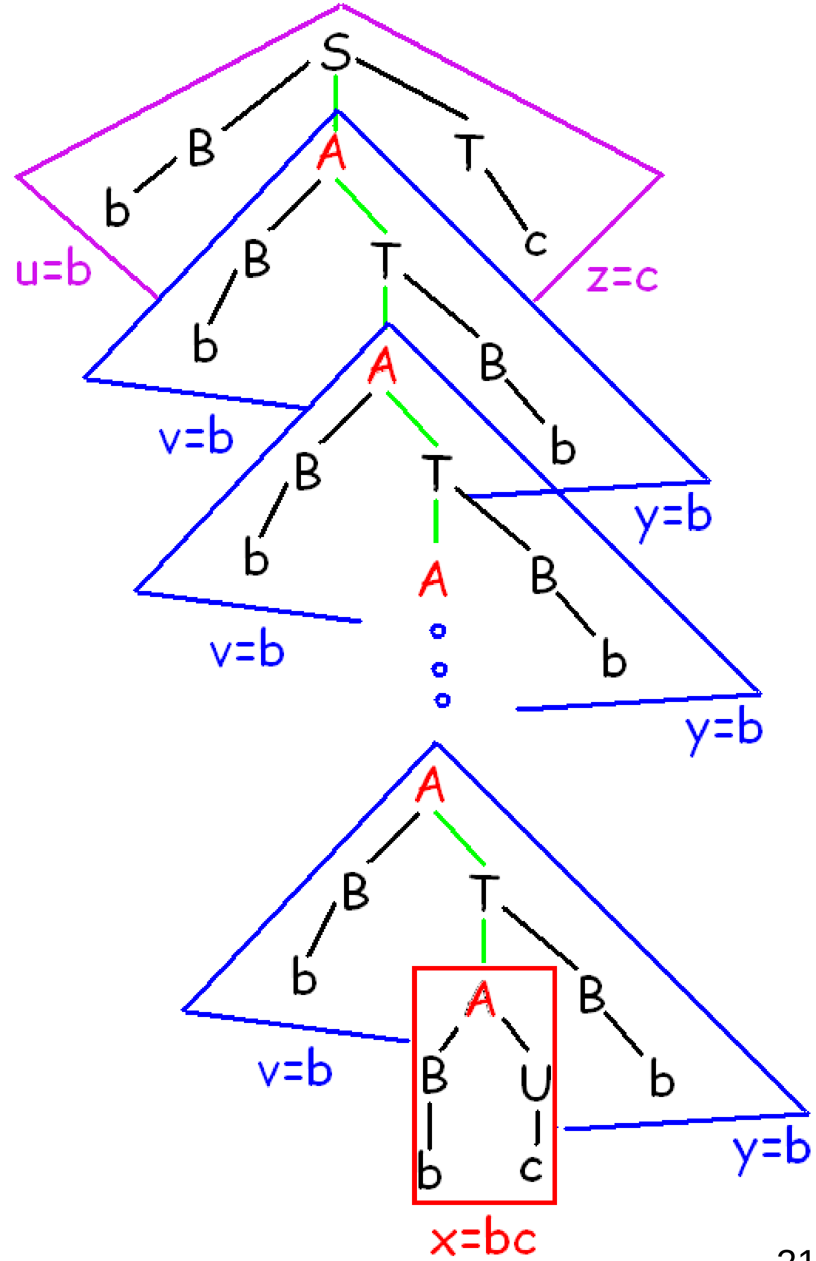
$= u\, v^3\, x\, y^3\, z$



To pump n times:

New yield is:

$= u\, v^n\, x\, y^n\, z$



The Pumping Theorem for Context-Free Languages:

Let G be a context-free grammar.

Then there exists some constant k which depends on G such that for any string w which is generated by G with $|w| \geq k$,

there exists u, v, x, y, z, such that

1. $w = u\, v\, x\, y\, z$,
2. $|v| + |y| \geq 1$, and
3. $u\, v^n\, x\, y^n\, z$ is in L for all $n \geq 0$.

$L = \{ a^n b^p : \quad n \le p \le 3n, \quad n,p \ge 0 \}$

Start symbol S.

$S \rightarrow a\,S\,b$ $\qquad$ $S \rightarrow \varepsilon$

$S \rightarrow a\,S\,bb$

$S \rightarrow a\,S\,bbb$

This works because any integer p can be expressed as:

$p = r + 2(n - r)$ when $n \le p \le 2n$, and

$p = 2r + 3(n - r)$ when $2n \le p \le 3n$.

Prove the following languages are context-free by designing context-free grammars which generate them:

$L_1 = \{a^n b^n c^p \;:\; n, p \geq 0\}$

$L_2 = \{a^n b^p c^n \;:\; n, p \geq 0\}$

$L_3 = \{a^n b^m : n \neq m, \;\; n, m \geq 0\}$

Hint: $L_3 = \{a^n b^m : n < m\} \cup \{a^n b^m : n > m\}$

$L_4 = \{\, c\,u\,c\,v\,c : |u| = |v|, \;\; u, v \in \{a, b\}^* \,\}$

What is $L_1 \cap L_2$?