Given the DFA below which accepts L, prove that  $L^{R} = \{ u^{R} : u \in L \}$  is regular by designing a NDFA which accepts  $L^{R}$ .

For example, abaa is in L so  $(abaa)^{R}$  = aaba is in L<sup>R</sup>.



## Announcements

The midterm is in class on Wed. June 21. There is a tutorial on Tuesday June 13. No tutorial on Tuesday June 20. Midterm tutorial: Monday June 19, 6:30pm, ECS 123. Bring any questions you have about assignments 1-3, old midterms or any other class material.

Assignment 3 is due on Friday at the beginning of class.

The preliminary final exam schedule has CSC 320 at 2pm on Monday August 14.

Let  $L = \{ a^n b^p : n+4 \le p \le 5n+7 \}$ 

1.What string w would you choose if you want to prove that L is not regular?

2. What are the cases for y?

3. How many times would you pump for each case?

4. If you chose instead w=  $a^r b^{2r}$  where all you assume is that  $k \le 3r$ , how does this change the proof?

The Pumping Lemma for Regular Languages: If L is a language accepted by a DFA with k states, and  $w \in L$ ,  $|w| \ge k$ , then there exists x, y, z such that

- 1. w = x y z,
- γ ≠ ε,
- 3.  $|xy| \le k$ , and
- 4.  $x y^n z$  is in L for all  $n \ge 0$ .

## Recipe for using the pumping lemma (proof by contradiction):

- 1.Assume L is regular and is accepted by a DFA M that has k states.
- 2. Choose w so that  $w \in L$  and  $|w| \ge k$ .
- Find all factorizations of w as w=xyz such that |xy|≤ k and y is not the empty string.
- 4. For each factorization of w, find n so that when you pump n times to get x y<sup>n</sup> z the resulting string is not in L. Make sure you look at the definition of L here and not just the pattern for w.

What machine  $M_2$  would your construction from assignment 2 create as input for isEmpty given this machine  $M_1$  and u=aaba:



Black box image from: http://socialcapitalmarkets.net/2013/04/03/whats-inside-the-socap13-black-box/<sup>6</sup>

regular expression	finite state machine	
a		Algorithms to Answer
a*		Questions about Regular
a+		Languages
a/b		if (if23==23)
(a/b)*c(d/e)		x= -23.2e23-6;
http://www.cgl.ucsf.edu/Outreach/bmi280/slides/swc/lec/re.html		7

The first step of a compiler is to break your program into tokens. Tokens:

- Keywords: if
- Brackets: ( )
- Variables: if 23 x
- Assignment: =
- Math Operator: -

if (if23==23) x= -23.2e23-6;

Logical: ==

Delimiter: ;

Double:-23.2e23

Integers: 23 6

Keywords:

if U while U int U double U switch U .... Variables: Not a Keyword but of the form: (a-z U A-Z)(a-z U A-Z U 0-9 U \_ )\* Non-negative Integers: N= (0 U (1-9)(0-9)\*) Numeric values:

 $(\Phi^* \cup -) \cup N (\Phi^* \cup . (0-9)^*)$  $(\Phi^* \cup (e \cup E)(\Phi^* \cup + \cup -) N)$  The pumping lemma or closure properties can be used to prove languages are not regular.

Regular languages are closed under:

- union
- concatenation
- Kleene star
- complement
- intersection

- exclusive or
- difference
- reversal

There are algorithms for the following questions about regular languages:

- 1. Given a DFA M and a string w, is  $w \in L(M)$ ?
- 2. Given a DFA M, is  $L(M) = \Phi$ ?
- 3. Given a DFA M, is  $L(M) = \Sigma^*$ ?
- 4. Given DFA's  $M_1$  and  $M_2$ , is  $L(M_1) \subseteq L(M_2)$ ?
- 5. Given DFA's  $M_1$  and  $M_2$ , is  $L(M_1) = L(M_2)$ ?

How can we use these to check correctness of student answers for the java tutorial?

## Java Regular expression tutorial:

S= student answer, T= teacher answer



Strings student generates but should not. Is S intersect the complement of  $T = \Phi$ ?

## Java Regular expression tutorial:

S= student answer, T= teacher answer



Strings student should generate but does not. Is T intersect the complement of S =  $\Phi$ ? The Pumping Lemma for Regular Languages:

- If L is a language accepted by a DFA with k states, and  $w \in L$ ,  $|w| \ge k$ , then  $\exists x, y, z$  such that
- 1. w = x y z,
- y ≠ ε,
- 3.  $|xy| \le k$ , and
- 4.  $x y^n z$  is in L for all  $n \ge 0$ .

The pumping lemma is NOT strong enough to work directly to prove that certain languages are not regular.

- Let L be a language which has a constant k such that for all  $w \in L$ ,  $|w| \ge k$ ,  $\exists x, y, z$  such that
- 1. w = x y z,This is necessary but2.  $y \neq \varepsilon$ ,not sufficient for a3.  $|xy| \le k$ , andregular.
- 4.  $x y^n z$  is in L for all  $n \ge 0$ .

Then you CANNOT conclude that L is regular.

Counterexample: See assignment 3.

 $L_1 = \{ u u^R v : u, v in \{0, 1\}^+ \}$