Given the DFA below which accepts $L$, prove that $L^{R}=\left\{u^{R}\right.$ : $u \in L\}$ is regular by designing a NDFA which accepts $L^{R}$.

For example, abaa is in $L$ so $(a b a a)^{R}=a a b a$ is in $L^{R}$.


## Announcements

The midterm is in class on Wed. June 21.
There is a tutorial on Tuesday June 13.
No tutorial on Tuesday June 20. Midterm tutorial: Monday June 19, 6:30pm, ECS 123. Bring any questions you have about assignments 1-3, old midterms or any other class material.

Assignment 3 is due on Friday at the beginning of class.

The preliminary final exam schedule has CSC 320 at 2pm on Monday August 14.

Let $L=\left\{a^{n} b^{p}: n+4 \leq p \leq 5 n+7\right\}$

1. What string $w$ would you choose if you want to prove that $L$ is not regular?
2. What are the cases for $y$ ?
3. How many times would you pump for each case?
4. If you chose instead $w=a^{r} b^{2 r}$ where all you assume is that $k \leq 3 r$, how does this change the proof?

## The Pumping Lemma for Regular Languages:

If $L$ is a language accepted by a DFA with $k$ states, and $w \in L,|w| \geq k$, then there exists $x, y, z$ such that

1. $w=x y z$,
2. $y \neq \varepsilon$,
3. $|x y| \leq k$, and
4. $x y^{n} z$ is in $L$ for all $n \geq 0$.

Recipe for using the pumping lemma (proof by contradiction):

1. Assume $L$ is regular and is accepted by a DFA $M$ that has $k$ states.
2. Choose $w$ so that $w \in L$ and $|w| \geq k$.
3. Find all factorizations of $w$ as $w=x y z$ such that $|x y| \leq k$ and $y$ is not the empty string.
4. For each factorization of $w$, find $n$ so that when you pump $n$ times to get $x y^{n} z$ the resulting string is not in $L$. Make sure you look at the definition of $L$ here and not just the pattern for $w$.

What machine $M_{2}$ would your construction from assignment 2 create as input for isEmpty given this machine $M_{1}$ and $u=a a b a$ :


Black box image from:
http://socialcapitalmarkets.net/2013/04/03/whats-inside-the-socap13-black-box/
regular expression
finite state machine

Algorithms to Answer Questions about Regular Languages
if (if23==23) $x=-23.2 e 23-6$;
$(\mathrm{a} / \mathrm{b})^{*} \mathrm{c}(\mathrm{d} / \mathrm{e})$


$a / b$


The first step of a compiler is to break your if (if23==23) $x=-23.2 e 23-6$; Tokens:

Keywords: if
Brackets: ( )
Logical: ==
Delimiter: ;
Variables: if23 x
Assignment: =
Math Operator: - Integers: 236

Double:-23.2e23

Keywords:
if $U$ while $U$ int $U$ double $U$ switch $U$....
Variables: Not a Keyword but of the form:
$(a-z \cup A-Z)\left(a-z \cup A-Z \cup 0-9 \cup Z_{-}\right)$
Non-negative Integers: $\mathrm{N}=\left(0 \cup(1-9)(0-9)^{\star}\right)$
Numeric values:
( $\left.\Phi^{\star} \cup-\right) \cup N\left(\Phi^{\star} \cup .(0-9)^{\star}\right)$
$\left(\Phi^{\star} \cup(e \cup E)\left(\Phi^{\star} U+U-\right) N\right)$

The pumping lemma or closure properties can be used to prove languages are not regular.

Regular languages are closed under:

- union
- concatenation
- Kleene star
- complement
- intersection
- exclusive or
- difference
- reversal

There are algorithms for the following questions about regular languages:

1. Given a DFA $M$ and a string $w$, is $w \in L(M)$ ?
2. Given a DFA $M$, is $L(M)=\Phi$ ?
3. Given a DFA $M$, is $L(M)=\Sigma^{*}$ ?
4. Given DFA's $M_{1}$ and $M_{2}$, is $L\left(M_{1}\right) \subseteq L\left(M_{2}\right)$ ?
5. Given DFA's $M_{1}$ and $M_{2}$, is $L\left(M_{1}\right)=L\left(M_{2}\right)$ ?

How can we use these to check correctness of student answers for the java tutorial?

Java Regular expression tutorial:
$S=$ student answer, $T=$ teacher answer


Strings student generates but should not.
Is $S$ intersect the complement of $T=\Phi$ ?

## Java Regular expression tutorial:

$S=$ student answer, $T=$ teacher answer


Strings student should generate but does not.
Is $T$ intersect the complement of $S=\Phi$ ?

The Pumping Lemma for Regular Languages:
If $L$ is a language accepted by a DFA with $k$ states, and $w \in L,|w| \geq k$, then $\exists x, y, z$ such that

1. $w=x y z$,
2. $y \neq \varepsilon$,
3. $|x y| \leq k$, and
4. $x y^{n} z$ is in $L$ for all $n \geq 0$.

The pumping lemma is NOT strong enough to work directly to prove that certain languages are not regular.

Let $L$ be a language which has a constant $k$ such that for all $w \in L,|w| \geq k, \exists x, y, z$ such that

1. $w=x y z$,
2. $y \neq \varepsilon$,
3. $|x y| \leq k$, and
This is necessary but not sufficient for a language to be regular.
4. $x y^{n} z$ is in $L$ for all $n \geq 0$.

Then you CANNOT conclude that $L$ is regular.
Counterexample: See assignment 3.
$L_{1}=\left\{u u^{R} v: u, v\right.$ in $\left.\{0,1\}^{+}\right\}$

