$$
\begin{array}{ll}
L_{1}=\{\varepsilon, 0,1,00,01,10,11\} & L_{3}=\{ \}=\Phi \\
L_{2}=\{0\} & L_{4}=\{\varepsilon\}
\end{array}
$$

The alphabet is $\Sigma=\{0,1\}$.
What are the languages:

1. $\overline{L_{1}}$
2. $\mathrm{L}_{2} \cup \mathrm{~L}_{3}$
3. $\mathrm{L}_{2} \mathrm{UL}_{4}$
4. $\mathrm{L}_{1} \cdot \mathrm{~L}_{1}$
5. $\mathrm{L}_{3} \cdot \mathrm{~L}_{2}$
6. $\mathrm{L}_{1} \cap \mathrm{~L}_{3}$
7. $\mathrm{L}_{4} \cdot \mathrm{~L}_{2}$

## CO-OP \&-CAREER

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## CO-OP + CAREER <br> Mock Interview Clinic

For engineering and computer science students

Practice your interview skills with a REAL co-op employer

Get immediate, on-the-spot feedback


Mock interviews will take place on Wednesday, May 31 from 9 a.m. -4:30 p.m. in the SUB Upper Lounge.

To register, visit learninginmotion.uvic.ca | Registration deadline: Friday, May 19.

Operations on Languages:

1. Complement of $L$ defined over $\Sigma=\bar{L}$
$=\left\{w \in \Sigma^{*}: w\right.$ is not in $\left.L\right\}$
2. Concatenation of Languages $L_{1} \cdot L_{2}=L_{1} L_{2}=$
$\left\{w=x \cdot y\right.$ for some $x \in L_{1}$ and $\left.y \in L_{2}\right\}$
3. Kleene star of $L, L^{*}=\left\{w=w_{1} w_{2} w_{3} \ldots w_{k}\right.$ for some $k \geq 0$ and $w_{1}, w_{2}, w_{3}, \ldots, w_{k}$ are all in $\left.L\right\}$
4. $L^{+}=L \cdot L^{*}$
(Concatenate together one or more strings from L.)

Matrix multiplication:


Concatenation:
$a b \cdot b b=a b b b$
$b b \cdot a b=b b a b$

$$
\begin{array}{ll}
L_{1}=\{\varepsilon, 0,1,00,01,10,11\} & L_{3}=\{ \}=\Phi \\
L_{2}=\{0\} & L_{4}=\{\varepsilon\}
\end{array}
$$

What are the languages:

1. $\mathrm{L}_{1}{ }^{*}$
2. $\mathrm{L}_{2} \cdot\left(\mathrm{~L}_{1}{ }^{*}\right)$
3. $\mathrm{L}_{2}{ }^{*}$
4. $\left(L_{2} \cdot L_{1}\right)^{*}$
5. $\mathrm{L}_{3}{ }^{*}$
6. $\mathrm{L}_{2} \cdot\left(\mathrm{~L}_{1}{ }^{*}\right) \cdot \mathrm{L}_{2}$
7. $\mathrm{L}_{4}{ }^{*}$

## Precedence of Operators

## Exponents

highest
Multiplication

Addition


Concatenation

Union

$$
\begin{array}{ll}
L_{1}=\{\varepsilon, 0,1,00,01,10,11\} & L_{3}=\{ \}=\Phi \\
L_{2}=\{0\} & L_{4}=\{\varepsilon\}
\end{array}
$$

What are the languages:

1. $\mathrm{L}_{4} \mathrm{U}\left(\mathrm{L}_{3} \cdot \mathrm{~L}_{2}\right)$
2. $\left(\mathrm{L}_{4} \mathrm{U} \mathrm{L}_{3}\right) \cdot \mathrm{L}_{2}$

What does $\mathrm{L}_{4} \mathrm{U} \mathrm{L}_{3} \cdot \mathrm{~L}_{2}$ mean?
How is this interpreted (add parentheses)? $\{a\} \cdot\{b\} \cup\{a\} \cdot\{b\}^{*} \cdot\{a\}$
$L_{2}=\left\{w \in\{0,1\}^{*}: w\right.$ is the binary representation of a prime with no leading zeroes\}

The complement is:
$\left\{w \in\{0,1\}^{\star}: w\right.$ is the binary representation of a number which is not prime which has no leading O's or w starts with 0\}

Note: 1 is not prime or composite. The string 1 is in the complement since it is not in $L$.

Regular Languages over Alphabet $\Sigma$ :
[Basis] 1. $\Phi$ and $\{\sigma\}$ for each $\sigma \in \Sigma$ are regular languages.
[Inductive step] If $L_{1}$ and $L_{2}$ are regular languages, then so are:
2. $L_{1} \cdot L_{2}$,
3. $L_{1} \cup L_{2}$, and
4. $L_{1}{ }^{*}$.

## Regular Languages over Alphabet $\Sigma$ :

[Basis] 1. $\Phi$ and $\{\sigma\}$ for each $\sigma \in \Sigma$ are regular languages.
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3. $L_{1} \cup L_{2}$, and
4. $L_{1}{ }^{*}$.

Example:
$\left\{w \in\{a, b\}^{*}: w\right.$ contains
aab as a substring\}

## Regular expressions over $\Sigma$ :

[Basis] 1. $\Phi$ and $\sigma$ for each $\sigma \in \Sigma$ are regular expressions.
[Inductive step] If $\alpha$ and $\beta$ are regular expressions, then so are:
2. ( $\alpha \beta$ )
3. ( $\alpha \cup \beta$ ) and
4. $a^{\star}$

Note: Regular expressions are strings over
$\Sigma \cup\left\{(),, \Phi, U,{ }^{*}\right\}$
for some alphabet $\Sigma$.

Prove the following languages over $\Sigma=\{0,1\}$ are regular by giving regular expressions for them:

1. $\{w: w$ has odd length\}
2. $\{w: w$ contains 0011$\}$
3. $\{w: w$ does not contain 01\}
4. $\{w: w$ starts and ends with the same symbol $(|w| \geq 1)\}$

TUTORIAL: $L=\left\{w\right.$ an element of $\{a, b\}^{*}: w$ has both baa and aaba as a substring \}.
$(a \mid b)^{\star} b a a(a \mid b)^{\star} a a b a(a \mid b)^{\star} \mid(a \mid b)^{\star} a a b a(a \mid b)^{\star} b a a(a \mid b)^{\star}$


Your Answer


| Your answer should generate the following strings but does not |
| :---: | :---: | :---: | :---: |
| aabaa, baaba, aaabaa, aabaaa, aabaab, abaaba, baaaba, baabaa, |
| baabab, bbaaba, aaaabaa, aaabaaa, aaabaab, aabaaaa, aabaaab, |
| aabaaba, aabaabb, abaaba, abaabaa, abaabab |


|  |  |  |  |
| :---: | :---: | :---: | :---: |
| Lesson | Syntax | Hint | Answer |
| Previous Question | Next Question |  | Submit |

MISSING: aabaa, baaba,

See home page for link to regular expression tutorial

