Let S =

{ π : π is a permutation of {1, 2, 3, …, n}

for some integer $n \geq 1$ }.

(a) List the elements of S for n= 1, 2, and 3.

(b) Prove that the set S is countable by explicitly describing a bijection between S and the natural numbers.

(c) How does your bijection number the permutations you listed for part (a)?

CSC 320:

Mathematics of computation

Α - Alpha (Al-fah)	Ν - Nu
Β - Beta (Bay-tah)	Ξ - Xi (Zie)
Γ - Gamma	Ο - Omicron
Δ - Delta	Π - Pi (pie)
Ε - Epsilon	Ρ - Rho (row)
Ζ - Zeta	Σ - Sigma
Η - Eta (ate-ah)	Τ - Tau (tahw)
Θ - Theta	Υ - Upsilon
Ι - Iota (I -oh-ta)	Φ - Phi (fie)
Κ - Kappa	Χ - Chi (kie)
Λ - Lamda	Ψ - Psi (sigh)
Μ - Mu	Ω - Omega

Mathematics of Computation

Introduction to the mathematics of formal language theory:

1. Alphabets, strings, languages.
2. Mathematical operations on strings: concatenation, substring, prefix, suffix, reversal.
3. Union, concatenation and Kleene star for languages.

These definitions lead up to our first class of languages- the regular languages.

Review: Representing Data

Alphabet: finite set of symbols

Ex. { a, b, c, … , z}

String: finite sequence of alphabet symbols

Ex. abaab, hello, cccc

Inputs and outputs of computations:

represented by strings.

ε represents an empty string (length 0)

Empty Set and Empty String

Students frequently are confused about the empty set and an empty string.

Empty set = Φ = { } = set with 0 elements.

Empty string= ε = `` = string with 0 symbols.

Example:

The set {ε} is a set containing one string.

Language: set of strings

First names of students taking CSC 320:

{Abdulaziz, Abdulmajeed, Abdulrahman, Addie, Alex, Aria, Behnam, Bowei, Bradley, Brandon, Brendon, Cameron, Casey, Chad, Chris, Christina, Cole, Derrick, Dhaimil, Dylan, Ellie, Eric, Erik, Geoff, Graeme, Hayley, Himmat, Ian, Jake, Jason, Jeremy, JianZhao, Jiaquan, Jingjing, Jodie, Jonathan, Jordan, Jose, Justin, Kai, Kaitlin, Keifer, Kelvin, Kira, Kun, Leo, Liam, Lingyao, Lisa, Lok, Louis, Maston, Matt, Matthew, Maxwell, Meagan, Morgan, Nicola, Noah, Omnielle, Paul, Quintan, Rafael, Reed, Rhiannon, Rich, Richard, Rui, Sanja, Sean, Shane, Shawn, Shiyi, Siting, Sonia, Tania, Taylor, Terance, Tim, Tony, Tristan, Tyler, William, Yihe, Yuanfan, Yves, Zhaoxuan, Zirui}

Notational Conventions

x, y, z real numbers

k, n integers

A, B matrices

p, q primes

$\Sigma, \Sigma', \Sigma_1 \Sigma_2$ alphabets

a, b, c, 0, 1 symbols

u, v, w, x, y, z strings

L, L_1, L_2 languages

Concatenation of strings x and y denoted $x \cdot y$ or simply xy means write down x followed by y.

Theorem: For any string w, $\varepsilon \cdot w = w = w \cdot \varepsilon$.

String v is a prefix of w if w= v y for some string y.

String v is a suffix of w if w= x v for some string x.

String v is a substring of w if there are strings x and y such that w= x v y.

The reversal of string w, denoted w^R, is w "spelled backwards".

Σ^* = set of all strings over alphabet Σ

Language over Σ – any subset of Σ^*

Examples: $\Sigma = \{0, 1\}$

$L_1 = \{ w \in \Sigma^* : w$ has an even number of 0's$\}$

$L_2 = \{ w \in \Sigma^* : w$ is the binary representation of a prime number with no leading zeroes$\}$

$L_3 = \Sigma^*$

$L_4 = \{ \} = \Phi$

$L_5 = \{ \varepsilon \}$

Operations on Languages:

1. Complement of L defined over Σ = $\overline{L}$

   = { w $\in \Sigma^*$: w is not in L }

2. Concatenation of Languages $L_1 \cdot L_2 = L_1 L_2$ =

   {w= x·y for some x $\in L_1$ and y $\in L_2$}

3. Kleene star of L, L^* = { w= $w_1 w_2 w_3 \ldots w_k$ for some k ≥ 0 and $w_1, w_2, w_3, \ldots, w_k$ are all in L}

4. $L^+ = L \cdot L^*$

(Concatenate together one or more strings from L.)

L_2= {w $\in$ {0,1}* : w is the binary representation of a prime with no leading zeroes}

The complement is:

{w $\in$ {0,1}* : w is the binary representation of a number which is not prime which has no leading 0's or w starts with 0}

Note: 1 is not prime or composite. The string 1 is in the complement since it is not in L.

Matrix multiplication:

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}$$

Concatenation:

ab · bb = abbb

bb · ab = bbab