1.(a) State the pumping lemma for regular languages.

(b) Let $w = a^k\, b\, a^{k^2}$. Describe all possible ways of choosing x, y, z such that $w = xyz$, and $y \neq \varepsilon$.

(c) Apply the pumping lemma to $w = a^k\, b\, a^{k^2}$ to prove that $L = \{a^n\, b\, a^m : n^2 \leq m \leq n^3\}$ is not accepted by a DFA with $k^2 + k + 1$ states.

(d) A more judicious choice for w would have made the argument for (c) much simpler. Suggest a better choice for w. How does this simplify the argument you gave for (c)?

2. Use the algorithm described in class (or in the text) to create a DFA equivalent to the FA with start state s_1 and transitions:

State	String	Next state
s_1	ε	s_3
s_1	bb	s_4
s_2	a	s_1
s_3	a	s_2
s_3	ε	s_2
s_4	ε	s_3

The set of final states includes only s_2.

3.(a) Give a context-free grammar for $L_1 = \{\, a^n\ b\ a^n\ b\ a^k : n, k \geq 0\}$.

(b) Design a PDA for $L_2 = \{\, a^n\ b\ a^k\ b\ a^n : n, k \geq 0\}$.

(c) Given that $L_3 = \{\, a^n\ b\ a^n\ b\ a^n : n \geq 0\}$ is not a context-free language, what can you say about closure of context-free languages under intersection?