Name: _____

ID Number:_____

CSC 320 Midterm Exam

Wed. June 27, 2012

Instructions:

- 1. Put your name on every page of the exam.
- 2. No calculators or other aids. Closed book.
- 3. Read through the entire exam before beginning. You should have 8 pages including this header page.
- 4. If you need more space you can write on the backs of the pages.

Question	Value	Mark
1	40	
2	20	
3	20	
4	20	
Total	100	

1. For parts (a), (b), (c) and (d) below, you must choose four DIFFERENT languages from the five given here and are required to find a regular expression, a context-free grammar, a DFA, and a PDA for them respectively. Choose carefully to minimize your effort.

 $L_{1} = \{ a^{p} b^{q} c^{r} d^{s} : (p+q) = (r+s), \quad p, q, r, s \ge 0 \}$ $L_{2} = \{ w \in \{0,1\}^{*} : w \text{ has both } 0 \ 1 \ 0 \ 1 \ and \ 0 \ 1 \ 1 \ 0 \ as \ substrings } \}$ $L_{3} = \{ w \in \{a,b\}^{*} : w \text{ has } aab \ as \ a \ prefix \ and \ baa \ as \ a \ suffix } \}$ $L_{4} = \{ a^{p} c^{q} a^{r} : p \ne q, \ p \ne r, \ q \ne r, \ and \quad p, q, r \ge 0 \}$ $L_{5} = \{ w \in \{\phi, *, a, b, (,), \cup \}^{*} : w \ represents \ a \ regular \ expression \ as \ given \ by \ the \ formal \ definition \ of \ a \ regular \ expression } \}$

Fill in your choices for each part:

Part	Requirement	Language chosen
(a)	Regular Expression	
(b)	Context-free Grammar	
(c)	Deterministic Finite Automaton	
(d)	Pushdown Automata	

(a) [10 marks] Give a regular expression for one of the languages.Your choice of language is:

(b) [10 marks] Give a context-free grammar for one of the languages.Your choice of language is:

[Question #1, continued]

(c) [10 marks] Draw the transition diagram of a DFA for one of the languages (include comments). **Your choice of language is:**

(d) [10 marks] Design a PDA which accepts one of the languages. Your choice of language is:

Final states:

State	Read	Рор	Next State	Push

- -4-
- 2. [20 marks] Use the construction described in class (which is the same as the one in the text) to convert this NDFA to an equivalent DFA:



State	Symbol	Q	Next state

Start state: _____

Final states: _____

A picture of your final DFA:

- 3. Let $L = \{ (ab)^n c^m : n \le m \le 3n \text{ and } n, m \ge 0 \}$. A proof that *L* is not regular starts by assuming that *L* is accepted by some DFA *M* that has *k* states. Let $w = (ab)^p c^{2p}$ where *p* has been chosen such that $2p \ge k$.
- (a) [8 marks] Describe all possible ways of choosing x, y, z such that w = x y z, $|xy| \le k$ and $y \ne \varepsilon$. Use as many cases as you need.

Case	X	У	Z	Conditions
1				
2				
3				
4				
5				
6				
7				

(b) [4 marks] For which cases from part (a) can you finish the proof of this case by pumping 0 times? Show the resulting string for each of these cases and explain why it is not in *L*.

[Question #3, continued]

For this question:

$$L = \{ (ab)^n c^m : n \le m \le 3n \text{ and } n, m \ge 0 \}.$$
$$w = (ab)^p c^{2p} \text{ where } 2p \ge k.$$

(c) [8 marks] Finish the proof for the cases from (a) that cannot be completed by pumping 0 times.

- 4. Circle **True** or **False** and justify your answer. **No marks will be given unless** there is a correct justification.
- (a) [5 marks] If $x \notin L_1$ and $y \notin L_2$ then $x \cdot y \notin L_1 \cdot L_2$. True False

 (b) [5 marks] A regular language can contain a subset which is not a regular language. True False

(c) [5 marks] The set ϕ^* does not contain any strings. True False

(d) [5 marks] If $L = \{ w \in \{ a, b \}^* : w = a^n b^n, n \ge 0 \}$, then $\bar{L} = \{ w \in \{ a, b \}^* : w = a^n b^m, n > m \}$ $\cup \{ w \in \{ a, b \}^* : w = a^n b^m, n < m \}$. True False Use this page if you need more space.

Clearly indicate the question you are answering.