Name:

## ID Number:

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## CSC 320 Midterm Exam

Wed. June 27, 2012

## Instructions:

1. Put your name on every page of the exam.
2. No calculators or other aids. Closed book.
3. Read through the entire exam before beginning. You should have 8 pages including this header page.
4. If you need more space you can write on the backs of the pages.

| Question | Value | Mark |
| :---: | :---: | :---: |
| 1 | 40 |  |
| 2 | 20 |  |
| 3 | 20 |  |
| 4 | 20 |  |
| Total | $\mathbf{1 0 0}$ |  |

1. For parts (a), (b), (c) and (d) below, you must choose four DIFFERENT languages from the five given here and are required to find a regular expression, a context-free grammar, a DFA, and a PDA for them respectively. Choose carefully to minimize your effort.
$L_{1}=\left\{a^{p} b^{q} c^{r} d^{s}:(p+q)=(r+s), \quad p, q, r, s \geq 0\right\}$
$L_{2}=\left\{w \in\{0,1\}^{*}: w\right.$ has both 0101 and 0110 as substrings $\}$
$L_{3}=\left\{w \in\{a, b\}^{*}: w\right.$ has aab as a prefix and baa as a suffix $\}$
$L_{4}=\left\{a^{p} c^{q} a^{r}: p \neq q, p \neq r, q \neq r\right.$, and $\left.p, q, r \geq 0\right\}$
$L_{5}=\left\{w \in\left\{\phi,{ }^{*}, a, b,(,), \cup\right\}^{*}: w\right.$ represents a regular expression as given by the formal definition of a regular expression \}
Fill in your choices for each part:

| Part | Requirement | Language chosen |
| :--- | :--- | :--- |
| (a) | Regular Expression |  |
| (b) | Context-free Grammar |  |
| (c) | Deterministic Finite Automaton |  |
| (d) | Pushdown Automata |  |

(a) [10 marks] Give a regular expression for one of the languages. Your choice of language is:
(b) [10 marks] Give a context-free grammar for one of the languages. Your choice of language is:
[Question \#1, continued]
(c) [10 marks] Draw the transition diagram of a DFA for one of the languages (include comments). Your choice of language is:
(d) [10 marks] Design a PDA which accepts one of the languages. Your choice of language is:
Start state:
Final states:

| State | Read | Pop | Next State | Push |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
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|  |  |  |  |  |

2. [20 marks] Use the construction described in class (which is the same as the one in the text) to convert this NDFA to an equivalent DFA:


| State | Symbol | Q | Next state |
| :--- | :--- | :--- | :--- |
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|  |  |  |  |

Start state: $\qquad$
Final states: $\qquad$
A picture of your final DFA:
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3. Let $L=\left\{(a b)^{n} c^{m}: n \leq m \leq 3 n\right.$ and $\left.n, m \geq 0\right\}$. A proof that $L$ is not regular starts by assuming that $L$ is accepted by some DFA $M$ that has $k$ states.
Let $w=(a b)^{p} c^{2 p}$ where $p$ has been chosen such that $2 p \geq k$.
(a) [8 marks] Describe all possible ways of choosing $x, y, z$ such that $w=x y z$, $|x y| \leq k$ and $y \neq \varepsilon$. Use as many cases as you need.

| Case | x | y |  | z |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  | Conditions |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |
| 7 |  |  |  |  |

(b) [4 marks] For which cases from part (a) can you finish the proof of this case by pumping 0 times? Show the resulting string for each of these cases and explain why it is not in $L$.

## [Question \#3, continued]

For this question:

$$
\begin{aligned}
& L=\left\{(a b)^{n} c^{m}: n \leq m \leq 3 n \text { and } n, m \geq 0\right\} \\
& w=(a b)^{p} c^{2 p} \text { where } 2 p \geq k
\end{aligned}
$$

(c) [8 marks] Finish the proof for the cases from (a) that cannot be completed by pumping 0 times.
4. Circle True or False and justify your answer. No marks will be given unless there is a correct justification.
(a) [5 marks] If $x \notin L_{1}$ and $y \notin L_{2}$ then $x \cdot y \notin L_{1} \cdot L_{2}$. True False
(b) [5 marks] A regular language can contain a subset which is not a regular language. True False
(c) [5 marks] The set $\phi^{*}$ does not contain any strings. True
False
(d) [5 marks] If $L=\left\{w \in\{a, b\}^{*}: w=a^{n} b^{n}, n \geq 0\right\}$, then $\bar{L}=\left\{w \in\{a, b\}^{*}: w=a^{n} b^{m}, n>m\right\}$
$\cup\left\{w \in\{a, b\}^{*}: w=a^{n} b^{m}, n<m\right\}$.
True
False
-8-
Use this page if you need more space.
Clearly indicate the question you are answering.

