## CSC 320 Midterm Exam

June 20, 2008

## Instructions:

1. Put your name on every page of the exam.
2. No calculators or other aids. Closed book.
3. Read through the entire exam before beginning. You should have 7 pages including this header page.

| Question | Value | Mark |
| :---: | :---: | :---: |
| 1 | 30 |  |
| 2 | 25 |  |
| 3 | 25 |  |
| 4 | 20 |  |
| Total | $\mathbf{1 0 0}$ |  |

Name: $\qquad$

ID Number:
1.(a) [10 marks] Prove that the language
$L=\left\{w \in\{0,1\}^{*}: w\right.$ has 01 as a prefix and 10 as a suffix $\}$
is regular by designing a DFA which accepts $L$.
(b) [10 marks] Prove that the language
$L=\left\{w \in\{a, b\}^{*}: w\right.$ contains both $a a b$ and $a b a$ as substrings $\}$ is regular by giving a regular expression which generates $L$.
(c) [10 marks] Design a nondeterministic finite automaton which accepts $L=(01 \cup 011 \cup 00)^{*} 011$
2. [25 marks] Use the construction described in class (which is the same as the one in the text) to convert this NDFA to an equivalent DFA:

| State | Symbol | Q | Next state |
| :--- | :--- | :--- | :--- |
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|  |  |  |  |

Start state: $\qquad$
Final states: $\qquad$
A picture of your final DFA:
3.(a) [5 marks] State the pumping lemma for regular languages.
(b) [5 marks] Let $w=a^{r} b a^{3 r}$. Describe all possible ways of choosing $x, y, z$ such that $w=x y z$, and $y \neq \varepsilon$.
(c) $[10$ marks $]$ Apply the pumping lemma to $w=a^{r} b a^{3 r}$ to prove that $L=\left\{a^{n} b a^{m}: n \leq m \leq 3 n\right\}$ is not accepted by a DFA with $4 r+1$ states.
(d) [5 marks] A more judicious choice for $w$ would have made the argument for (c) much simpler. Suggest a better choice for $w$. How does this simplify the argument you gave for (c)?
4. Circle True or False and justify your answer. No marks will be given unless there is a correct justification.
(a) [5 marks] If $x \notin L_{1}$ and $y \notin L_{2}$ then $x y \notin L_{1} L_{2}$. True

False
(b) [5 marks] A regular language can contain a subset which is not a regular language. True

False
(c) [5 marks] The set $\phi^{*}$ does not contain any strings. True

False
(d) [5 marks] The language $L=\left\{u u^{R} v: u, v \in\{a, b\}^{+}\right\}$is regular because for any string $w$ of length at least four, $w$ can be factored as $x y z$ where $|x y|$ is at most four and $x y^{n} z$ is in $L$ for all $n \geq 0$.

True
False

Use this page if you need more space.
Clearly indicate the question you are answering.

