Name: ______________________________

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UNIVERSITY OF VICTORIA
EXAMINATIONS- AUGUST 2010
CSC 320 - Foundations of Computer Science
Section A01, CRN 30202
Instructor: Wendy Myrvold
Duration: 3 hours

TO BE ANSWERED ON THE PAPER.

Instructions:

This question paper has eleven pages (the last page is blank in case you need extra space) plus the header page.

Students **MUST** count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

Use only space provided on exam for answering questions. Closed book. No aids permitted.

Question	Value	Mark
1	20	
2	10	
3	25	
4	15	
5	10	
6	20	
Total	**100**	

1. [20 marks] For each of the following languages, indicate the most restrictive of the classes below into which it falls

   (a) finite

   (b) regular

   (c) context-free

   (d) Turing-decidable

   (e) Turing-acceptable

   (f) None of the above.

Example:

$L = \{ a^n b^n : n \geq 0\}$ The correct answer is (c) since L is context-free, but is not regular.

_____ i) $\{ a^n b^m c^p : n \leq p \}$

_____ ii) $\{ w \in \{ a \}^* : |w|$ *is congruent to* 11 *or* 13 *mod* $23 \}$

_____ iii) The complement of $(a \cup b)^* (a \cup b)^*$

_____ iv) The complement of $\{ a^n b^n c^n : n \geq 0 \}$

_____ v) $\{ w w^R : w \in \{ a \}^* \}$

_____ vi) $\{u u^R v : u, v \in \{0, 1\}^+\}$

_____ vii) $\{ w w : w \in \{a, b\}^* \}$

_____ viii) $\{ w \in \{a, b, c\}^* : w$ *has at most one occurrence of* $a, b,$ *and* $c\}$

_____ ix) $\{ a^{n^2} : n \geq 0 \}$

_____ x) $\{ a^n b a^{n-5} : n \geq 5 \}$

For each of the following languages, indicate the most restrictive of the classes below into which it falls

(a) finite

(b) regular

(c) context-free

(d) Turing-decidable

(e) Turing-acceptable

(f) None of the above.

_____ xi) $\{\ w \in \{0,1\}^* :\ w \textit{ does not contain } 0\,1\,0\,1\,1 \textit{ as a substring } \}$

_____ xii) $\{w \in \{(,),,,\mathrm{a},\mathrm{b}\}^*$: w represents a correspondance system which has a match }.

_____ xiii) $\{u\ :\ u = {''M''} \textit{ for some TM } M \textit{ and } |u| \leq 100{,}000\}$

_____ xiv) $\{\ ({''M_1''}, {''M_2''}, {''w''})\ : \textit{both } M_1 \textit{ and } M_2 \textit{ accept input } w\ \}$

_____ xv) $\{\ {''M''}\ : M \textit{ halts on every string }\}$

_____ xvi) $\{\ ({''M''}, {''w''})\ : \textit{TM } M \textit{ does not halt when started on a blank tape }\}$

_____ xvii) $\{\ ({''M''}, {''w''})\ : \textit{TM } M \textit{ moves its head to the left on input } w\ \}$

_____ xviii) $\{\ ({''M''}, {''w''})\ : \textit{TM } M \textit{ accepts a regular language }\}$

_____ xix) $\{\ {''M''}\ : \textit{there is some string on which } M \textit{ halts }\}$

_____ xx) $\{\ {''D''}\ : D \textit{ is a DFA which accepts } \phi\ \}$

2. Draw the transition diagrams for DFA accepting the following two languages. Do not include any dead states.

(a) [2 marks] $L_1 = (aa \cup ab)^*$
Label the states $q_1, q_2, \cdots$

(b) [2 marks] $L_2 = (a \cup ba)^*$
Label the states $r_1, r_2, \cdots$

(c) [6 marks] Use the construction for proving that regular languages are closed under intersection to construct a DFA for $L_1 \cap L_2$.

3. Context-free languages.

(a) [5 marks] Give a context-free grammar that generates
$L_1 = \{w\ c\ w^R\ c\ u\ :\ u, w\ \in\ \{a, b\}^*\}$.

(b) [5 marks] Design a PDA that accepts
$L_2 = \{w\ c\ u\ c\ w^R\ :\ u, w\ \in\ \{a, b\}^*\}$.

$L_1 = \{w\ c\ w^R\ c\ u\ :\ u, w \in \{a, b\}^*\}.$

$L_2 = \{w\ c\ u\ c\ w^R\ :\ u, w \in \{a, b\}^*\}.$

(c) [10 marks] What is $L_1 \cap L_2$? Use the pumping lemma to prove that this language is not context-free.

(d) [5 marks] Prove that context-free languages are not closed under intersection.

4. Suppose that aliens from outer space have invented a new method of computing that is very different from the machines which currently exist on earth.

   Problem 1: Given a TM M_1, is there any string in $\{0, 1\}^*$ that M_1 halts on?

   Problem 2: Given a TM M_2, does M_2 halt on the input 1010?

   The aliens have programmed an algorithm which takes as input a Turing machine M_1 encoded as $''M_1''$ and determines the answer to Problem 1.

(a) [7 marks] Prove that it is possible to decide the answer to Problem 2 using the algorithm that the aliens have developed for Problem 1. If you create a new TM in your proof, give its machine schema.

(b) [3 marks] Does your answer from (a) prove OUTCOME 1 or 2:

1. If Problem 1 is not Turing-decidable then Problem 2 is not Turing-decidable.
2. If Problem 2 is not Turing-decidable then Problem 1 is not Turing-decidable.

(c) [5 marks] Give the machine schema for a Turing machine *M* that decides if its input is equal to 1010. That is, *M* decides the language {1010}.

5. Suppose the head instructions for a TM M are numbered as follows:

0	1	2	3	4	5	6	7	8	9
L	R	#	(	)	q	a	0	1	,

The string $''M''$ is:
(q01, a0010, q01, a0000), (q01, a0100, q10, a0000),
(q10, a1000, q10, a1000), (q10, a1001, q00, a0000)

(a) [4 marks] What is M? **Start state of M:**

State	Symbol	Next state	Head Instruction

(b) [6 marks] Is the question: *Does M from part (a) halt on input "M"?* decidable or not? You can assume that the problem: *Given M, does M halt when started on a blank tape?* is not decidable in formulating your answer.

6. A *vertex cover* of a graph G is a subset S of the vertices of the graph G such that each edge of G has at least one endpoint in S.

VERTEX COVER PROBLEM

INSTANCE: Graph $G = (V, E)$, integer k.

QUESTION: Does G have a vertex cover of order k?

(a) [10 marks] Prove that the vertex cover problem is in NP. For any algorithms you use, specify precisely the input format to the algorithm, and give detailed pseudocode for your algorithm.

(b) [7 marks] The proof that VERTEX COVER is NP-complete describes how to create a graph from a 3-SAT system such that the graph has a vertex cover of a certain size if and only if the 3-SAT problem has a satisfying truth assignment. Describe the general procedure for creating the graph, and apply it to the 3-SAT system: $\{ x_1, x_2, x_3\}, \{ x_1, \bar{x}_2, \bar{x}_3\}, \{ \bar{x}_1, \bar{x}_2, x_4\}$.

(c) [3 marks] What size vertex cover are you searching for in this graph when the 3-SAT problem has n variables $(x_{1,}\ x_2, \cdots x_n)$ and m clauses?

Use this page if you need extra space. Clearly indicate the question you are answering.

The End.