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UNIVERSITY OF VICTORIA
EXAMINATIONS- DECEMBER 2010
CSC 320 - Foundations of Computer Science
Section A01/A02, CRN 10877/10878
Instructor: Wendy Myrvold
Duration: 3 hours

TO BE ANSWERED ON THE PAPER.

Instructions:

This question paper has eleven pages (the last page is blank in case you need extra space) plus the header page.

Students **MUST** count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

Use only space provided on exam for answering questions. Closed book. No aids permitted.

Question	**Value**	**Mark**
1	20	
2	15	
3	10	
4	20	
5	10	
6	10	
7	15	
Total	**100**	

1. [20 marks] For each of the following languages, indicate the most restrictive of the classes below into which it falls
   (a) finite
   (b) regular
   (c) context-free
   (d) Turing-decidable
   (e) Turing-acceptable
   (f) None of the above.

Example:

$L = \{ a^n b^n : n \geq 0\}$ The correct answer is (c) since L is context-free, but is not regular.

_____ i) $\{ w : w$ *is the decimal notation for an even number with no leading zeroes* $\}$

_____ ii) $\{ w : w$ *is the decimal notation for a composite number with no leading zeroes* $\}$

_____ iii) $\{ w \in \{0, 1\}^* : w$ *has* 0 1 0 0 *and* 1 0 1 1 0 *as substrings* $\}$

_____ iv) $(a \cup b)^* \phi (a \cup b)^*$

_____ v) $\{ a^n b^{n^2} : n \geq 0\}$

_____ vi) $\{ a^n b^m c^p : n \leq m \leq 3n \text{ or } 2n \leq p \leq 5n\}$

_____ vii) $\{ a^n b^m c^p : n \leq m \leq 3n \text{ and } 2n \leq p \leq 5n\}$

_____ viii) $\{u u^R v : u, v \in \{0, 1\}^+\}$

_____ ix) $\{ w w : w \in \{a\}^* \}$

_____ x) $\{ w w : w \in \{a, b\}^* \}$

For each of the following languages, indicate the most restrictive of the classes below into which it falls

(a) finite

(b) regular

(c) context-free

(d) Turing-decidable

(e) Turing-acceptable

(f) None of the above.

_____ xi) { w : w *is a C program that given as input a TM* M *decides if* M *halts when started on a blank tape* }

_____ xii) { (M, w) *: TM M halts on input w using at most* $1,000,000$ *tape squares* }

_____ xiii) { (M, w) *: TM M does not halt on input w* }

_____ xiv) { (M, c) *: TM M prints the symbol c when started on a blank tape* }

_____ xv) { $''M''$ *: there is some string on which M halts* }

_____ xvi) { $''M''$ *: there are no strings on which M halts* }

_____ xvii) { $''M''$: M *is a TM* }

_____ xviii) { $''M''$ *: TM* M *accepts a finite language* }

_____ xix) { (M, w) *: TM M moves its head to the left on input w* }

_____ xx) { $''M''$ *: M writes a nonblank symbol when started on a blank tape* }

2. Circle true or false for each question and justify your answer. No marks will be given unless there is a correct justification.

(a) [5 marks] Context-free languages are closed under complement.
True **False**

(b) [5 marks] Turing-decidable languages are closed under complement.
True **False**

(c) [5 marks] For the pumping theorem for context-free languages, when *w* is factored as *uvxyz*, then *v* is never equal to the empty string.
True **False**

3. For an *empty-stack PDA* $M = (K, \Sigma, \Gamma, \Delta, s, F)$, $L(M) = \{w \in \Sigma^* : (s, w, e)$ *yields in zero or more steps* (f, e, e) *where* $f \in F\}$.

For a *stack-oblivious* PDA, $M = (K, \Sigma, \Gamma, \Delta, s, F)$, $L(M) = \{w \in \Sigma^* :$ (s, w, e) *yields in zero or more steps* (f, e, u) *where* $f \in F$ *and* $u \in \Gamma^*\}$.

(a) [4 marks] Describe a construction that will convert an arbitrary empty-stack PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$, to a stack-oblivious PDA $M = (K', \Sigma, \Gamma', \Delta', s', F')$, such that both PDA's accept the same language.

(b) [2 marks] What language does the following empty-stack PDA accept?
Start state: p, Final states: $\{q\}$

State	Read	Pop	Next State	Push
p	a	*e*	p	BB
p	a	*e*	p	BBB
p	*e*	*e*	q	*e*
q	b	B	q	*e*

(c) [4 marks] Apply your construction from (a) to the PDA from (b) to create an equivalent stack-oblivious PDA.

4. [20 marks] Apply the Pumping Theorem to $w = a^k\ b\ a^k\ b\ a^{2k}$ to prove that $L = \{a^p\ b\ a^q\ b\ a^r\ :\ p, q, r \geq 0,\ q \geq p,\ and\ r \leq 2p\}$ is not context-free.

5(a) [4 marks] State the definition of what it means for a TM $M = (K, \Sigma, \delta, s)$ to decide a language L defined over an alphabet Σ.

(b) [6 marks] Design a TM that decides the language $L = \{w \in \{0,1\}^* : w \textit{ starts with } 011\}$.

On this page, please list the names of the states that you will use and the meaning of being in each state. There is space for the transitions of your TM on the next page.

The transitions for your TM deciding $L = \{w \in \{0,1\}^* : w \textit{ starts with } 011\}$.

Start State:

State	Symbol	Next	Head	Comments

6. Suppose that a professor of CSC 320 using a newly invented quantum computer has written a program which takes as input a TM M_1 and answers

Question 1: "Does M_1 halt on input 10110?".

For your CSC 320 homework, you have created a TM M_2 and you want to know the answer to

Question 2: "Does M_2 halt on input *abaa*?".

(a) [7 marks] Prove that it is possible to decide the answer to Question 2 using the algorithm that the professor has developed for Question 1. If you create a new TM in your proof, give its machine schema.

(b) [3 marks] Does your answer from (a) prove OUTCOME 1 or 2:

1. If Question 1 is not Turing-decidable then Question 2 is not Turing-decidable.
2. If Question 2 is not Turing-decidable then Question 1 is not Turing-decidable.

7. A path that contains every vertex of a graph is a *Hamilton path*. A cycle that contains every vertex of a graph is a *Hamilton cycle*.

HAMILTON PATH PROBLEM

INSTANCE: Graph $G = (V, E)$.

QUESTION: Does G have a Hamilton path?

HAMILTON CYCLE PROBLEM

INSTANCE: Graph $G = (V, E)$.

QUESTION: Does G have a Hamilton cycle?

(a) [5 marks] Prove that if there is polynomial time algorithm for HAMILTON PATH then there is a polynomial time algorithm for HAMILTON CYCLE.

(b) [3 marks] What is the time complexity of your approach from part (a) assuming that the algorithm for HAMILTON PATH takes time $O(n^3 + m^2)$, where n is the number of vertices of the graph and m is the number of edges?

(c) [3 marks] Assuming that both HAMILTON PATH and HAMILTON CYCLE have been proved to be in NP, which of these statements have you proved with your answer to (a):
S1: If HAMILTON PATH is NP-complete then HAMILTON CYCLE is NP-complete.
S2: If HAMILTON CYCLE is NP-complete then HAMILTON PATH is NP-complete.

(d) [4 marks] Prove that HAMILTON PATH is in NP. Give pseudocode and analyse the time complexity of any algorithms that you develop.

Use this page if you need extra space. Clearly indicate the question you are answering.

The End.