Name: ______________________________

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UNIVERSITY OF VICTORIA

EXAMINATIONS- AUGUST 1998

CSC 320 - Foundations of Computer Science

Instructor: W. Myrvold

Time: 3 hours

TO BE ANSWERED ON THE PAPER.

Instructions:

Students **MUST** count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

This question paper has ten pages (the last page is blank in case you need extra space) plus the header page.

Use only space provided on exam for answering questions. Closed book. No aids permitted.

Question	**Value**	**Mark**
1	20	
2	15	
3	20	
4	15	
5	15	
6	15	
Total	**100**	

1. [20] For each of the following languages, indicate the most restrictive of the classes below into which it falls
   (a) finite
   (b) regular
   (c) context-free
   (d) Turing-decidable
   (e) Turing-acceptable
   (f) None of the above.

Example:

$L = \{ a^n b^n : n \geq 0 \}$ The correct answer is (c) since L is context-free, but is not regular.

In this question, we will use $''M''$ to denote the encoding of a Turing Machine M and $''w''$ to denote the encoding of the string w.

_____ i) $L = \{ w w^R : w \in \{ a \}^* \}$

_____ ii) $L = \{ w w^R : w \in \{ a , b \}^* \}$

_____ iii) $\{ w w : w \in \{a , b \}^* \}$

_____ iv) $L = (\phi)(a \cup b)^*$

_____ v) $\{ a^p : p \textit{ is a prime number} \}$

_____ vi) $L = \{ a^n b^m c^p : n = 2p \}$

_____ vii) $L = \{ w \in \{ 0,1 \}^* : w \textit{ does not contain } 01011 \textit{ as a substring} \}$

_____ viii) $L = \{ a^n b^n c^n : n \geq 0 \}$

_____ ix) $\{ \textit{strings in } \{ a , b\}^* \textit{ with an odd number of a's and and even number of b's} \}$

Question #1 (continued)

(a) finite
(b) regular
(c) context-free
(d) Turing-decidable
(e) Turing-acceptable
(f) None of the above.

_____ x) *{ strings in* $\{a, b\}^*$ *with length less than 93 }*

_____ xi) L = *the complement of* $\{ a^n b^n : n \geq 0 \}$

_____ xii) $L = \{ w c w c w : w \in \{a, b\}^* \}$

_____ xiii) *{ ″M″ ″w″ : M is a TM, w is an input string }*

_____ xiv) *{ ″M″ ″w″ : TM M accepts input w }*

_____ xv) *{ ″M″ ″w″ : TM M does not accept input w }*

_____ xvi) *{ ″D″ ″w″ : D is a DFA which accepts w}*

_____ xvii) *{ ″M″ ″a″ : TM M never prints an ”a” when started on a blank tape }*

_____ xviii) *{ ″M″ ″a″ : TM M prints the symbol a when started on a blank tape after computing for at most one billion steps }*

_____ xix) *{ ″M″ : M writes a nonblank symbol when started on a blank tape }*

_____ xx) *{ ″M″ : there is some string on which M halts }*

2. The *exclusive or* of two languages L_1 and L_2, denoted $L_1 \oplus L_2$, is defined to be $\{w : (w \in L_1 \textit{ or } w \in L_2) \textit{ and } w \textit{ is not in } L_1 \cap L_2\}$.

(a) [6] Prove that regular languages are closed under **exclusive or** by describing a construction for a DFA $M = (K, \Sigma, \delta, s, F)$ for $L_1 \oplus L_2$ given DFA's $M_1 = (K_1, \Sigma, \delta_1, s_1, F_1)$ and $M_2 = (K_2, \Sigma, \delta_2, s_2, F_2)$ for L_1 and L_2 respectively. Hint: a construction similar to the ones derived for union and intersection on assignment #2 works.

Question #2 (continued)

Draw the transition diagrams for DFA's accepting the following two languages:

(b) [2] $L_1 = (a \cup b)^* b$	**(c) [2]** $L_2 = (aa \cup ab \cup ba \cup bb)^*$
Label the states $q_1, q_2, \cdots$	**Label the states** $r_1, r_2, \cdots$

(d) [5] Show the results of your construction from (a) as applied to the DFA's you have given for parts (b) and (c).

Transition function		
State	**Symbol**	**Next state**
	a	
	b	
	a	
	b	
	a	
	b	
	a	
	b	
	a	
	b	

Transition Diagram for this DFA

3. Context-free languages.

(a) [3] Give a regular grammar for $L_1 = a^* \, b^* \, a^*$.

(b) [2] Is it possible to find a grammar for L_1 from part (a) which is not regular? Justify your answer.

(c) [5] Design a PDA for $L_2 = \{a^n \, b^m \, a^p : \; n, m, p \geq 0 \, , \, n \neq m\}$ without first constructing a grammar for the language. Include lots of comments.

State	Read	Pop	Next	Push	Comments

Question #3 (continued)

Let $L_1 = a^* b^* a^*$ (from (a)),

$L_2 = \{a^n b^m a^p : n, m, p \geq 0, n \neq m\}$ (from (b)) and

$L_3 = \{a^n b^m a^p : n, m, p \geq 0, n \neq m \text{ or } m \neq p \text{ or } n \neq p\}$.

(d) [8] What is $L_1 \oplus L_3$? State the pumping theorem and use it to prove that this language is not context-free.

(e) [2] Argue that context-free languages are not closed under $\oplus$.

4.(a) [5] Give the machine schema for a TM C which makes a copy of an input string w defined over $\Sigma = \{a, b\}$. Starting with # w [#] on the input tape, the final result should be # w # w [#]. Include lots of comments.

(b) [7] Suppose you are given a TM M_1 which when started with #w[#] halts with #Y[#] if w is in some language L_1 and #N[#] if w is not in L_1 and you also have such a machine for some other language L_2. Give the machine schema for a machine which works like this for $L_1 \oplus L_2$. Your schema may include the machine C you designed for (a).

(c) [3] What does part (b) tell you about closure under $\oplus$?

Question 5:

For this question, you may assume only that $K_0 = \{''M''\, ''w'' :\ \textit{TM M halts when started on input } w\}$ has been shown not to be Turing-decidable.

(a) [5] Let $K = \{''M'' :\ \textit{TM M halts when started on a blank tape}\}$. Is K Turing acceptable? Justify your answer.

(b) [5] Is K from part (a) Turing-decidable? Justify your answer.

(c) [5] Let $L_1 = \Sigma^*$ where Σ is the alphabet you are using to encode Turing machines. Define K as per part (b). What is $L_2 = L_1 \oplus K$? What (if anything) does this tell you about closure of Turing-acceptable languages under $\oplus$?

Question 6:

The problem 3_COLOURABLE takes as input a graph G on n vertices and m edges and returns **true** if the vertices of G can be coloured so that if two vertices are adjacent they are coloured with different colours and **false** otherwise.

(a) [5] Prove that 3-COLOURABLE is in NP.

(b) [5] Explain how you could solve 3-COLOURABLE using a function which solves 3-SAT.

(c) [5] Does your work for parts (a)-(b) prove that 3-COLOURABLE is NP-complete given that 3-SAT is NP-complete? If not, what would you have to do to prove it?

Use this page if you need extra space. Clearly indicate the question you are answering.

The End.