CSC 320 - Final Examination
Fall 1997

1. [20] For each of the following languages, indicate the most restrictive of the classes below into which it falls:
   (a) finite
   (b) regular
   (c) context-free
   (d) Turing-decidable
   (e) Turing-acceptable
   (f) None of the above.

Example:

$L = \{ a^n b^n : n \geq 0\}$ The correct answer is (c) since L is context-free, but is not regular.

In this question, we will use $\rho(M)$ to denote the encoding of a Turing Machine M and $\rho(w)$ to denote the encoding of the string w. Assume that the encoding scheme used is as described in class and in the text.

_____ i) { *strings in* $\{a, b\}^*$ *with an odd number of a's and and even number of b's* }

_____ ii) ϕ^*

_____ iii) $L =$ *the complement of* $\{ a^n b^n : n \geq 0 \}$

_____ iv) $\{ w w^R u u^R : u, w \in \{a, b\}^* \}$

_____ v) $\{ w w : w \in \{a, b\}^* \}$

_____ vi) $L = \{ w \in \{0, 1\}^* :$ w *does not contain* 0 1 0 1 1 *as a substring* }

_____ vii) { $\rho(M)\ \rho(w)$ *: TM M accepts a regular language* }

_____ viii) { a^p : *p is a prime number* }

_____ ix) $L =$ *the complement of* $\{ a^n b^n c^n : n \geq 0 \}$

_____ x) $L = \{ a^n b^m c^p : n = 2p \}$

_____ xi) $L = (\phi)(a \cup b)^*$

_____ xii) { *strings in* $\{a, b\}^*$ *which do not have aa, ab, ba or bb as a substring* }

_____ xiii) { $\rho(M)$ *: M writes a nonblank symbol when started on a blank tape* }

_____ xiv) { $\rho(M)\ \rho(a)$ *: TM M prints the symbol a when started on a blank tape* }

_____ xv) { $\rho(M)\ \rho(a)$ *: TM M has at least one transition on the symbol a* }

_____ xvi) { $\rho(M_1)\ \rho(M_2)\ \rho(w)$ *: both* M_1 *and* M_2 *accept input w* }

_____ xvii) { $\rho(M)\ \rho(w)$: *M is a TM, w is an input string* }

_____ xviii) { $\rho(M)\ \rho(w)$ *: TM M does not accept input w* }

_____ xix) { $\rho(M)$ *: M halts on every string* }

_____ xx) { $\rho(M)$ *: there is some string on which M halts* }

2. [12] Are the following languages closed under the given operations? Fill in the chart with *yes* or *no* as appropriate.

Class/Operation	Union	Intersection	Complement
Regular	(a)		
Context-free			
Turing-decidable			(b)
Turing-acceptable	(c)		

3.(a)[5] Given two DFA's $M_1 = (K_1, \Sigma, \delta_1, s_1, F_1)$ and $M_2 = (K_2, \Sigma, \delta_2, s_2, F_2)$, your aim is to construct a **DFA** $M = (K, \Sigma, \delta, s, F)$ which accepts the union of $L(M_1)$ and $L(M_2)$. Each state of M corresponds to a pair in $K_1 \times K_2$ and the significance of being in the state labelled (q, r) with $q \in K_1$ and $r \in K_2$ is that M_1 would be in state q and M_2 would be in state r after the same input.

(i) Which state should be the start state for M?

(ii) Which states should be final states?

(ii) Define the transition function δ of M in terms of δ_1 and δ_2.

(iii) What does this tell you about the answer for the box marked (a) in Question #2 and why?

(b) [5] Prove your answer to the box marked (b) in Question 2 (Are Turing-decidable languages closed under complement?).

(c) [5] Prove your answer to the box marked (c) in Question 2. (Are Turing-acceptable languages closed under union?).

4.(a) [5] State the pumping lemma for regular languages.

(b) [10] Prove that $\{c\ I^n\ c\ I^{n+1}\ c\ :\ n \geq 0\}$ is not regular using the pumping lemma.

5.(a) [5] Design a PDA for $L_1 = \{c\ I^n\ c\ I^k c\ I^{2n} c\ :\ n, k \geq 0\}$ without resorting to finding a grammar first. Include comments.

(b) [5] Give a context-free grammar for $L_2 = \{c\ I^m\ c\ I^k c\ I^{2n} c\ :\ n \leq k \leq 2n\,;\ n, m \geq 0\}$. Include comments which explain how your grammar works.

6. A *path* in a graph can be represented by a sequence of distinct vertices $v_1, v_2, \ldots\ v_k$, such that for i from 1 to $k-1$, (v_i, v_{i+1}) is an edge of the graph. A *cycle* can be represented similarly with the additional condition that edge (v_k, v_1) must be present.

LONGEST PATH PROBLEM	LONGEST CYCLE PROBLEM
Instance: Graph G, integer k	Instance: Graph G, integer k
Question: *Does G have a path with k or more vertices?*	Question: *Does G have a cycle with k or more vertices?*

(a) [5] Prove that LONGEST CYCLE is in NP.

(b) [5] Use the knowledge that LONGEST PATH is NP-complete to prove that LONGEST CYCLE is NP-complete.

7. Consider the following string $\rho(M_1) = \; c\,I^2\,c \;\; c\,I^2 \;\; c\,I^3 \;\; c\,I^2 \;\; c\,I^1\,c \;\; c\,I^2 \;\; c\,I^4 \;\; c\,I^3 \;\; c\,I^1\,c \;\; c\,I^2 \;\; c\,I^5 \;\; c\,I^2 \;\; c\,I^1\,c \;\; c\,I^3 \;\; c\,I^3 \;\; c\,I^1 \;\; c\,I^2\,c \;\; c\,I^3 \;\; c\,I^4 \;\; c\,I^2 \;\; c\,I^1\,c \;\; c\,I^3 \;\; c\,I^5 \;\; c\,I^3 \;\; c\,I^1\,c\,c$

(a) [5] What is M_1? Fill in the following chart. Use symbols $\{\#, I, c, a, b, \cdots\}$ as needed.

Start state:			
State	Symbol	Next State	Head Instr.

(b) [5] Is it possible to decide whether M_1 from part (a) halts on input $\rho(M_1)$? Justify your claim.

8. For parts (a) and (b), suppose you know that the problem:
"Given TM M_1, does M_1 halt when started on a blank tape?"
is not Turing-decidable and further, it is the only problem you know to not be Turing-decidable.

(a) [5] Is the following problem Turing-decidable?
"Given TM M_2, does M_2 ever hang without writing any nonblank symbols when started on a blank tape?"

Circle **yes** or **no**. Prove your answer is correct.

(b) [5] Is the following problem Turing-decidable?
"Given TM M_3, does M_3 ever hang when started on some input w?"

Circle **yes** or **no**. Prove your answer is correct.