CSC 320 - Final Examination
Fall 1996

1. [20] For each of the following languages, indicate the most restrictive of the classes below into which it falls

   (a) finite

   (b) regular

   (c) context-free

   (d) Turing-decidable

   (e) Turing-acceptable

   (f) None of the above.

Example:

$L = \{ a^n b^n : n \geq 0 \}$ The correct answer is (c) since L is context-free, but is not regular.

In this question, we will use $\rho(M)$ to denote the encoding of a Turing Machine M and $\rho(w)$ to denote the encoding of the string w.

_____ i) *{ strings in* $\{ a , b\}^*$ *with an odd number of a's and and even number of b's }*

_____ ii) $L = (\phi)(a \cup b)^*$

_____ iii) $L = \{ w w^R : w \in \{ a \}^* \}$

_____ iv) $L = \{ w w^R : w \in \{ a , b \}^* \}$

_____ v) $L =$ *the complement of* $\{ a^n b^n : n \geq 0 \}$ over $\Sigma = \{a, b\}$

_____ vi) $L = \{ w \in \{0, 1\}^* : w$ *does not contain* 01011 *as a substring* $\}$

_____ vii) $L = \{ a^n b^m c^p : n = 2p \}$

(a) finite

(b) regular

(c) context-free

(d) Turing-decidable

(e) Turing-acceptable

(f) None of the above.

______ viii) $L = \{ a^n b^n c^n : n \geq 0 \}$

______ ix) $\{ \rho(M)\ \rho(w)$: *TM M accepts input w* $\}$

______ x) $\{ \rho(M)\ \rho(w)$: *TM M does not accept input w* $\}$

______ xi) $\{ \rho(M)\ \rho(w)$: *M is a TM, w is an input string* $\}$

______ xii) $\{ \rho(M)\ \rho(w)\ \rho(q)$: *TM M does not enter state q on input w* $\}$

______ xiii) $\{ \rho(M)$: *M halts on every string* $\}$

______ xiv) $\{ \rho(M)$: *there is some string on which M halts* $\}$

______ xv) $\{ \rho(M_1)\ \rho(M_2)\ \rho(w)$: *both* M_1 *and* M_2 *accept input w* $\}$

______ xvi) $\{ \rho(M)$: *TM M halts when started on a blank tape* $\}$

______ xvii) $\{ \rho(M)\ \rho(a)$: *TM M has at least one transition on the symbol a* $\}$

______ xviii) $\{ a^p : p$ *is a prime number* $\}$

______ xix) $\{ w\ w : w \in \{a, b\}^* \}$

______ xx) $L = \{ w : w$ *is the name of a student writing this exam* $\}$

2.(a) [4] State the pumping lemma for regular languages.

(b) [6] Prove that $\{w\,w \ : w \ \in \ \{a, b\}^*\}$ is not regular using the pumping lemma. Hint: Intersect with $a^* \, b \, a^* \, b$ first.

3.(a) [5] Design a PDA for $L_1 \ = \ \{a^n \, b^k \, c^{2n} \ : \ n, k \geq 0\}$.

(b) [5] Give a context-free grammar for $L_2 \ = \ \{a^n \, b^k \, c^m \ : \ n \leq k \leq 2n\,; \, n, m \geq 0\}$.

(c) [5] Suppose I proved that $L_3 \ = \ \{a^n \, b^k \, c^{2n} \ : \ n \leq k \leq 2n\,; \, n \geq 0\}$ is not context-free. What could you then deduce about the closure properties of context-free languages?

4. Consider the context-free grammar G with start symbol S, and rules: $S \rightarrow A\,T\,B$, $T \rightarrow B\,U\,A$, $U \rightarrow T\,B$, $T \rightarrow A$, $A \rightarrow a\,a$, $A \rightarrow a$, $B \rightarrow b\,b$, and $B \rightarrow b$.

(a) [4] Give a derivation in G for $w = a \ a \ b \ b \ a \ a \ b \ a \ a \ b$. For full marks, give a derivation which is not leftmost.

(b) [6] Draw the parse tree from your derivation from (a).

(c) [5] Use your parse tree and the ideas in the proof of the pumping theorem to find an infinite language which is contained in the language generated from the grammar in part (a).

(d) [5] Indicate on your parse tree from (b) the sections on the tree which correspond to u, v, x, y, z of the pumping theorem.

5(a). [4] Define what it means for a Turing Machine to *decide* a language.

(b) [6] Give the transitions for a TM which decides
$L = \{w \in \{a, b\}^* : w \textit{ has an odd number of } a's\}$.

Start state:			
State	Symbol	Next State	Head Instr.

(c) [5] Show the computation of your machine on $w = a\,b\,a$.

There was more space in this table on the previous final exam (omitted to save paper).

6.(a) [4] Prove that the HAMILTONIAN PATH PROBLEM is in NP.

(b) [6] Given that the HAMILTONIAN CYCLE PROBLEM is NP-complete, prove that the HAMILTONIAN PATH PROBLEM is NP-complete.

7. For this question, assume that the only language you know to not be Turing-decidable is $K_0 = \{ \rho(M)\ \rho(w) : M \textit{ halts on input } w\}$.

(a) [5] Let $M = (\{s\}, \{\#, a\}, \delta, s)$ where δ is defined by: $\delta(s, \#) = (s, R)$, and $\delta(s, a) = (h, a)$. Let $w = a\,a$. Give $\rho(M)\ \rho(w)$ where the symbols are ordered as $\#, a$.

(b) [5] Is it possible to decide if your string $\rho(M)\ \rho(w)$ from (a) is in K_0? Justify your answer.

(b) [5] Is it possible to decide if an arbitrary TM M halts on input $w = a\,a$? Justify your answer.