CSC 320- Practice Final Exam
Summer 1993

1. For each of the following languages, indicate the most restrictive of the classes below into which it falls
   (a) finite
   (b) regular
   (c) context-free
   (d) Turing-decidable
   (e) Turing-acceptable
   (f) None of the above.

Example:

$L = \{ a^n b^n : n \geq 0 \}$ The correct answer is (c) since L is context-free, but is not regular.

In this question, we will use $\rho(M)$ to denote the encoding of a Turing Machine M and $\rho(w)$ to denote the encoding of the string w. Assume that the encoding scheme used is as described in class and in the text.

_____ i) *The complement of* $(aa \cup bb)^* (a \cup b)^*$

_____ ii) $\{ a^{n^2} : n \geq 0 \}$

_____ iii) $\{ w\,w : w \in \{a, b\}^* \}$

_____ iv) $\{ \rho(M)\ \rho(w) :$ *M is a TM, w is an input string* $\}$

_____ v) $\{ \rho(M)\ \rho(w)$ *: TM M accepts input w* $\}$

_____ vi) $\{ \rho(M)\ \rho(w)$ *: TM M does not accept input w* $\}$

_____ vii) $\{ \rho(D)\ \rho(w)$ *: D is a DFA which accepts w*$\}$

_____ viii) $\{ \rho(D)$ *: D is a DFA which accepts some finite language* $\}$

_____ ix) $\{ \rho(D)$ *: D is a DFA which accepts some language which is not regular* $\}$

_____ x) $\{ \rho(M)\ \rho(a)$ *: TM M has at least one transition on the symbol a* $\}$

_____ xi) $\{ \rho(M)\ \rho(a)$ *: TM M prints the symbol a when started on a blank tape* $\}$

_____ xii) $\{\ \rho(M)\ \rho(a)$ *: TM M prints the symbol a when started on a blank tape after computing for at most one billion steps* $\}$

_____ xiii) $\{\ \rho(M)\ \rho(w)$ *: M is a deterministic TM which halts on w after an even number of moves* $\}$

_____ xiv) $\{$ *strings in* $\{\ a\ ,\ b\}^*$ *with length less than 13* $\}$

_____ xv) $a^*\ b^* \cup b^*\ a^*$

_____ xvi) ϕ

_____ xvii) $\{\ w\ w^R\ u\ u^R :\quad u\ ,\ w\ \in \{a\ ,\ b\}^*\ \}$

_____ xviii) $\{\ w\ w\ u\ u :\quad u\ ,\ w\ \in \{a\ ,\ b\}^*\ \}$

_____ xxix) $\{\ a^p$: *p is a prime number* $\}$

_____ xx) $\{$ *w : w is the unary notation for* $10^k\ \}$

2. Draw the transition diagrams for DFA accepting the following two languages. Do not include any dead states.

(a) $L_1\ =\ (\ aa \cup ab\ )^*$ **Label the states** $q_1, q_2, \cdots$

(b) $L_2\ =\ (\ a \cup ba\ )^*$ **Label the states** $r_1, r_2, \cdots$

(c) Use the construction for proving that regular languages are closed under intersection to construct a DFA for $L_1\ \cap\ L_2$.

3. Context-free languages.

(a) Give a PDA for $L_1\ =\ \{\ w\ c\ u\ c\ w^R\ :\ u, w\ \in \{a\ ,\ b\}^*\ \}$.

(b) Give a context-free grammar for $L_2\ =\ \{\ w\ c\ w^R\ c\ u\ :\ u, w\ \in \{a\ ,\ b\}^*\ \}$.

(c) What is $L_1 \cap L_2$? Use the pumping lemma to prove that this language is not context-free.

(d) Prove that context-free languages are not closed under intersection.

4. Turing Machines

(a) Design a TM M_L which when started on input $w \in \{ a, b \}^*$, positions the tape head one square to the right of the blank at the lefthand end of w. Give the transition function and a machine schema for M_L.

(b) Consider the following TM:

Does this TM ever write a ''*c*'' when started on input

i)	*abba* ?	yes	no	(Circle the correct response)
ii)	*aba* ?	yes	no	(Circle the correct response)
iii)	*abab* ?	yes	no	(Circle the correct response)

(c) What language over $\Sigma_0 = \{a , b \}$ does the TM from part (b) accept?

5. Which of the following problems are undecidable? Justify your claims.

(a) Is there any input on which the TM of question 4 part (b) writes a ''*c*''?

(b) Given some arbitrary string $w \in \{a , b \}^*$, does the TM of question 4 part (b) ever write a ''*c*'' while computing on input *w*?

(c) Will an arbitrary TM *M* ever write the symbol ''*c*'' when started on a blank tape?

6. The Hamilton Path and Cycle Problems were stated on the exam. Please refer to the study aid.

(a) Given that the Hamiltonian Path Problem is NP-complete, prove that the Hamiltonian Cycle Problem is NP-complete.
Hint: Deleting any edge from a Hamiltonian cycle gives a Hamiltonian path.

(b) Suppose I told you that I could solve the Hamiltonian cycle problem in $O(n^3)$ time where n is the number of vertices in the graph. What is the significance of this?