Name: ______________________________

ID Number: ___________________________

UNIVERSITY OF VICTORIA
EXAMINATIONS- DECEMBER 2009
CSC 320 - Foundations of Computer Science
Sections A01 (CRN 10867) and A02 (CRN 10868)
Instructor: Wendy Myrvold
Duration: 3 hours

TO BE ANSWERED ON THE PAPER.

Instructions:

Students **MUST** count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

This exam has ten pages (the last page is blank in case you need extra space) plus the header page.

Use only space provided on exam for answering questions. Closed book. No aids permitted.

Question	Value	Mark
1	20	
2	15	
3	5	
4	10	
5	10	
6	10	
7	15	
8	15	
Total	**100**	

1. [20 marks] For each of the following languages, indicate the most restrictive of the classes below into which it falls

   (a) finite

   (b) regular

   (c) context-free

   (d) Turing-decidable

   (e) Turing-acceptable

   (f) None of the above.

Example:

$L = \{ a^n b^n : n \geq 0\}$ The correct answer is (c) since L is context-free, but is not regular.

_____ i) $\{u\, u^R\, v \,:\, u, v \in \{0\}^+\}$

_____ ii) $\{u\, u^R\, v \,:\, u, v \in \{0, 1\}^+\}$

_____ iii) $\{ w \in \{ a \}^* : |w| \textit{ is congruent to } 11 \textit{ or } 13 \textit{ mod } 23 \}$

_____ iv) *The complement of* $\{ a^n b^n : n \geq 0 \}$

_____ v) *The complement of* $\{ a^n b^n c^n : n \geq 0 \}$

_____ vi) $\{u\, 1\, v\, c\, 1^k : |u| = k - 1, u, v \in \{0, 1\}^+\}$

_____ vii) $\{$ *strings in* $\{a, b\}^*$ *which do not have aa, ab, ba or bb as a substring* $\}$

_____ viii) $\{ w : w$ *is the unary notation for* $10^k \}$

_____ ix) $(a \cup b)^* \phi (a \cup b)^*$

_____ x) $\{ ''D'' : D$ *is a DFA which accepts the language* $\phi \}$

For each of the following languages, indicate the most restrictive of the classes below into which it falls

(a) finite

(b) regular

(c) context-free

(d) Turing-decidable

(e) Turing-acceptable

(f) None of the above.

_____ xi) $\{w \in \{(,),,,a\}^*$: w represents a correspondance system with a match }

_____ xii) $\{w \in \{(,),,,a,b\}^*$: w represents a correspondance system with a match }

_____ xiii) { $''M''$ *: M halts on every string* }

_____ xiv) { (M, a) *: TM M prints the symbol a when started on a blank tape after computing for at most one billion steps* }

_____ xv) { $''M''$ *: M writes a nonblank symbol when started on a blank tape* }

_____ xvi) { (M, w) *: TM M moves its head to the left on input w* }

_____ xvii) { $''M''$ *: there is some string on which M halts* }

_____ xviii) { (M, a) *: TM M has at least one transition on the symbol a* }

_____ xix) { (M_1, M_2, w) *: both M_1 and M_2 accept input w*}

_____ xx) { (M, w) *: TM M does not halt on input w* }

2.(a) [4] Give a DFA that accepts
$L_1 = \{w \in \{a, b\}^* :$ the number of $a's$ in w is odd.$\}$.

Start state:	
Final states:	

State	Symbol	Next State

(b) [4] Give a PDA that accepts
$L_2 = \{a^n b^m = n \leq m \leq 2n\}$.

Use different state names from (a).

Start state:	
Final states:	

State	Sym.	Pop	State	Push

(c) [7] Prove that $L_1 \cap L_2$ is context-free by applying the construction we learned in class (which is the same as the one in the text).

Start state:	
Final states:	

State	Sym.	Pop	State	Push

3. [5] Indicate on the parse tree a valid choice for *u*, *v*, *x*, *y* and *z* for the pumping theorem for context-free languages.

4. [10] What languages over $\Sigma = \{a, b\}$ do the following TM's accept?

5. [10] Let $L = \{w \in \{a, b\}^* : w = a^p\ b\ a^q\ b\ a^r \textit{ where } q \geq p \textit{ and } r \leq p\}$.

A CSC 320 student is trying to prove that this language is not context-free, and one of the cases is this one: $w = a^i\ a^j\ a^{k-i-j}\ b\ a^s\ a^t\ a^{k-s-t}\ b\ a^k$ where $v = a^j$ and $y = a^t$. The student writes: ‘‘Pump zero times and the string is not in L because there is less a’s in the first block than in the second one.’’ The TA being draconian with respect to part marks for incorrect or incomplete solutions gives the student zero for this answer. Give a correct proof for this case.

6. Suppose the head instructions for a TM M are numbered as follows:

0	1	2	3	4	5	6	7	8	9
L	R	#	(	)	q	a	0	1	,

The string $''M''$ is:
$(q01, a0010, q01, a0000), (q01, a0100, q10, a0000), (q10, a0111, q10, a0000),$
$(q10, a1000, q10, a0000), (q10, a0101, q11, a0101), (q10, a0110, q00, a0000),$
$(q11, a0101, q11, a0101)$

(a) [4] What is M? **Start state of M:**

State	Symbol	Next state	Head Instruction

(b) [6] Is the question: *Does M from part (a) halt on input "M"?* decidable or not? You can assume that the problem: *Given M, does M halt when started on a blank tape?* is not decidable in formulating your answer.

7. Arthur Dent, using space age technology not yet available on earth, developed an algorithm which determines if a TM M_1 halts or not when started on a blank tape. Then later, he discovered that the meaning to life, the universe, and everything is 42 and hence, 42 is an important input.

(a) [5] Given a TM M_2, prove that Arthur can determine if M_2 halts or not on the input 42 using the program he already developed which determines if a TM M_1 halts or not when started on a blank tape. If you create a new TM in your proof, give its machine schema.

(b) [5] Suppose there is a program which is faster than Arthur's but it answers the question of whether a TM M_2 halts on input 42. Explain how Arthur can use this algorithm to determine if some TM M_3 halts when started on a blank tape. If you create a new TM in your proof, give its machine schema.

(c) [5] We proved in class that that the problem of determining if a TM M halts when started on a blank tape is not decidable. Is it part (a) or part (b) which can be used to prove that it is also undecidable to determine if a TM M halts on input 42?

8. The problem 3-COLOURABLE takes as input a graph G on n vertices and m edges. It returns **true** if the vertices of G can be coloured using three colours so that if two vertices are adjacent they are coloured with different colours, and **false** otherwise.

(a) [5] Prove that 3-COLOURABLE is in NP.

(b) [5] Explain how you could solve 3-COLOURABLE using an algorithm which solves 3-SAT (each clause has at most 3 literals).

(c) [5] Does your work for parts (a)-(b) prove that 3-COLOURABLE is NP-complete given that 3-SAT is NP-complete? If not, what would you have to do to prove it?

Use this page if you need extra space. Clearly indicate the question you are answering.

The End.