Name: ______________________________

ID Number: ___________________________

UNIVERSITY OF VICTORIA
EXAMINATIONS- AUGUST 2008
CSC 320 - Foundations of Computer Science
Section A01
Instructor: Wendy Myrvold
Duration: 3 hours

TO BE ANSWERED ON THE PAPER.

Instructions:

Students **MUST** count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

This question paper has ten pages (the last page is blank in case you need extra space) plus the header page.

Use only space provided on exam for answering questions. Closed book. No aids permitted.

Question	**Value**	**Mark**
1	20	
2	15	
3	10	
4	10	
5	15	
6	15	
7	15	
Total	**100**	

1. [20 marks] For each of the following languages, indicate the most restrictive of the classes below into which it falls
   (a) finite
   (b) regular
   (c) context-free
   (d) Turing-decidable
   (e) Turing-acceptable
   (f) None of the above.

Example:

$L = \{ a^n b^n : n \geq 0\}$ The correct answer is (c) since L is context-free, but is not regular.

______ i) $\{ a^{n^2} : n \geq 0 \}$

______ ii) $\{ w \in \{1\}^* : w$ is the unary notation for the integer $10^k, k \geq 0\}$

______ iii) $\{ w \in \{0,1\}^* : w$ is the decimal notation for the integer $10^i, i \geq 0 \}$

______ iv) $\phi\ (a \cup b)^*$

______ v) $\{ a^n b^n c^n : n \geq 0 \}$

______ vi) $L =$ the complement of $\{ a^n b^n c^n : n \geq 0 \}$

______ vii) $\{ w \in \{a, b, c\}^* : w$ has at most one occurrence of $a, b,$ and $c\}$

______ viii) $\{u u^R v : u, v \in \{0\}^+\}$

______ ix) $\{u u^R v : u, v \in \{0, 1\}^+\}$

______ x) $\{ w \in \{0,1\}^* : w$ does not contain 00101 as a substring $\}$

For each of the following languages, indicate the most restrictive of the classes below into which it falls

(a) finite

(b) regular

(c) context-free

(d) Turing-decidable (recursive)

(e) Turing-acceptable (recursively enumerable)

(f) None of the above.

_____ xi) $\{ w \in \{a, b\}^* : |w|$ is congruent to 7 *or* 11 mod 23 $\}$

_____ xii) $\{ w \in \{a, b\}^* : |w| \leq 23\}$

_____ xiii) $\{u : u = "M"$ for some TM M and $|u| \leq 100{,}000\}$

_____ xiv) $\{"M" | "a"$: TM M prints the symbol a when started on a blank tape $\}$

_____ xv) $\{ "M"$: M writes a nonblank symbol when started on a blank tape$\}$

_____ xvi) $\{ "M_1" | "M_2" | "w"$: both M_1 and M_2 accept input w $\}$

_____ xvii) $\{ "M"$: there is some string on which M halts $\}$

_____ xviii) $\{ "M" | "w"$: TM M moves its head to the left on input w $\}$

_____ xix) $\{ "M" | "w"$: TM M does not halt on input w $\}$

_____ xx) $\{ "M" | "a"$: TM M has at least one transition on the symbol a $\}$

2. Let $L = \{w \in \{0,1\}^* :$ *w has an even number of* $0'$*s and the number of 1's in w is not divisible by* $3\}$.

(a) [5 marks] Design a DFA which accepts the language L.

(b) [5 marks] Give a regular context-free grammar which generates L.

(c) [5 marks] Answer true or false and justify your answer: All context-free grammars which generate L are regular context-free grammars because L is a regular language.

3. Let $L = \{w \in \{a, b\}^* : \text{ every prefix of } w \text{ has at least as many } a's \text{ as } b's\}$.

(a) [3 marks] List all strings of length at most three which are in L.

(b) [7 marks] Design a PDA which accepts the language L.

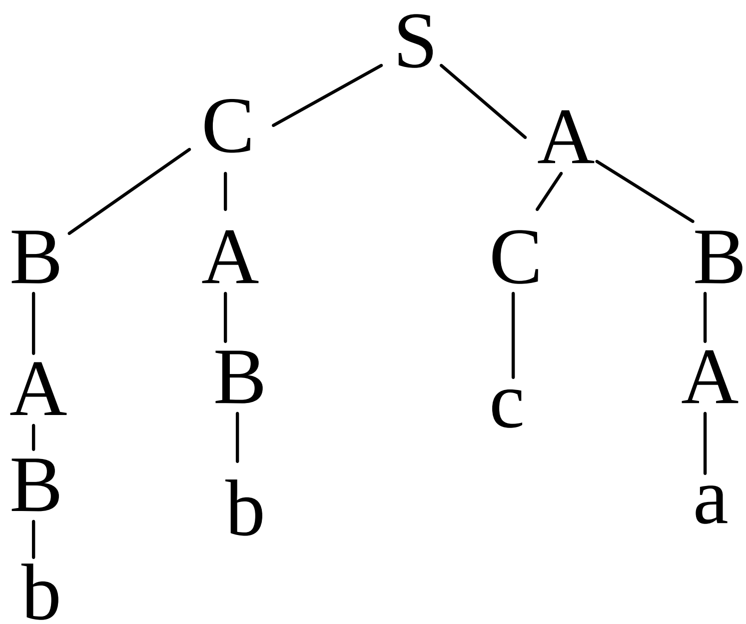
Start state:

Final states:

Transition function:

State	Symbol	Pop	Next state	Push	Comments

4. Consider the following parse tree for a context-free grammar:



(a) [5 marks] Which strings correspond to u, v, x, y and z of the pumping theorem ?

(b) [5 marks] Use the ideas from the pumping theorem to find an infinite language which is contained in the language generated from the grammar for this question.

5. Let $L = \{a^p\, b\, a^q\, b\, a^r : p \le q \le r\}$.

(a) [10 marks] Use the pumping theorem for context-free languages applied to the string $w = a^n\, b\, a^n\, b\, a^n$ to prove that the language L is not context-free.

(b) [5 marks] Answer true or false and justify your answer: For the proof for part (a), you can select $w = a^n ba^n ba^{n+1}$ instead of $w = a^n ba^n ba^n$.

6. Arthur Dent, using space age technology not yet available on earth developed an algorithm which determines if a TM M_1 halts or not when started on a blank tape. But then later, he discovered that the meaning to life, the universe, and everything is 42.

(a) [5 marks] Given a TM M_2, prove that Arthur can determine if M_2 halts or not on the input 42 using the program he already developed which determines if a TM M_1 halts or not when started on a blank tape. If you create a new TM in your proof, give its machine schema.

(b) [5 marks] Suppose there is a program which is faster than Arthur's but it answers the question of whether a TM M_2 halts on input 42. Explain how Arthur can use this algorithm to determine if some TM M_1 halts when started on a blank tape. If you create a new TM in your proof, give its machine schema.

(c) [5 marks] We proved in class that that the problem of determining if a TM M halts when started on a blank tape is not decidable. Is it part (a) or part (b) which can be used to prove that it is also undecidable to determine if a TM M halts on input 42?

7. A path that contains every vertex of a graph is a *Hamiltonian path*. A cycle that contains every vertex of a graph is *Hamiltonian cycle*.

HAMILTON PATH PROBLEM

INSTANCE: Graph $G = (V, E)$.

QUESTION: Does G have a Hamiltonian path?

HAMILTON CYCLE PROBLEM

INSTANCE: Graph $G = (V, E)$.

QUESTION: Does G have a Hamiltonian cycle?

(a) [5 marks] Prove that if there is polynomial time algorithm for HAMILTON PATH then there is a polynomial time algorithm for HAMILTON CYCLE.

(b) [3 marks] Analyze the time complexity of your approach from part (a) assuming that the algorithm for HAMILTON PATH takes time $O(n^3)$, where n is the number of vertices of the graph and m is the number of edges.

(c) [3 marks] Assuming that both HAMILTON PATH and HAMILTON CYCLE have been proved to be in NP, which of these statements have you proved with your answer to (a):
S1: If HAMILTON PATH is NP-complete then HAMILTON CYCLE is NP-complete.
S2: If HAMILTON CYCLE is NP-complete then HAMILTON PATH is NP-complete.

(d) [4 marks] Prove that HAMILTON PATH is in NP.

Use this page if you need extra space. Clearly indicate the question you are answering.

The End.