**Name:** ______________________________

**ID Number:** ____________________________

UNIVERSITY OF VICTORIA

EXAMINATIONS- AUGUST 2003

CSC 320 - Foundations of Computer Science

Instructor: W. Myrvold

Duration: 3 hours

TO BE ANSWERED ON THE PAPER.

**Instructions:**

Students **MUST** count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

This question paper has ten pages (the last page is blank in case you need extra space) plus the header page.

Use only space provided on exam for answering questions. Closed book. No aids permitted.

| **Question** | **Value** | **Mark** |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 10 | |
| 4 | 20 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| **Total** | **100** | |

1. [20 marks] For each of the following languages, indicate the most restrictive of the classes below into which it falls
   - (a) finite
   - (b) regular
   - (c) context-free
   - (d) Turing-decidable
   - (e) Turing-acceptable
   - (f) None of the above.

**Example:**

$L = \{ a^n b^n : n \geq 0 \}$ The correct answer is (c) since $L$ is context-free, but is not regular.

In this question, we will use $''M''$ to denote the encoding of a Turing Machine $M$ and $''w''$ to denote the encoding of the string $w$.

_____ i) { $w$ : $w$ is the unary notation for $10^k$ }

_____ ii) { *"M" : TM M* accepts a context-free language }

_____ iii) { *"M" " w "* : *TM M* accepts input $w$ }

_____ iv) { strings in $\{0, 1\}^*$ with $4r+3$ *0's* and $5s+2$ *1's*, $r, s \geq 0$ }

_____ v) { *"M" "w"* : *TM M* does not accept input $w$ }

_____ vi) $L = (a \cup b)^* \phi (a \cup b)^*$

_____ vii) { *"M"* : there is some string on which $M$ halts }

_____ viii) $L = \{ w \in \{0,1\}^* : w$ does not contain 0 1 0 1 1 as a substring $\}$

_____ ix) $(a \cup b)^* - (a^* \cup b^*)$

**Question #1 (continued)**

(a) finite

(b) regular

(c) context-free

(d) Turing-decidable

(e) Turing-acceptable

(f) None of the above.

_____ x) $\phi^*$

_____ xi) { "$M$" "$w$" : $M$ is a TM, $w$ is an input string }

_____ xii) { "$M_1$" "$M_2$" " $w$" : both $M_1$ *and* $M_2$ accept input $w$ }

_____ xiii) {"$M$" : $M$ halts on every string }

_____ xiv) $L = \{ a^n b^m c^p : n = 2p \}$

_____ xv) $L = \{ a^n b^n c^n : n \geq 0 \}$

_____ xvi) {"$M$" such that "$M$" has length at most 1500 }

_____ xvii) $L =$ the complement of $\{ a^n b^n c^n : n \geq 0 \}$

_____ xviii) {"$M$" "$w$" : *TM* $M$ moves its head to the left on input $w$ }

_____ xix) { $u$ : $u$ is a regular expression for some language defined over $\{a, b\}$ }.

_____ xx) {"$M$" such that "$M$" halts after at most 1500 steps}

2. Circle **True** or **False** and justify your answer. **No marks will be given unless there is a correct justification.**

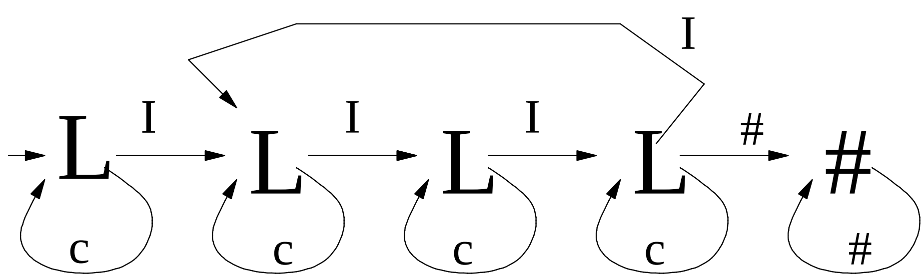
   (a) [5 marks] Consider the context free grammar which has start symbol $S$ and rules:

| | | |
|---|---|---|
| $S \rightarrow a\ S\ a$ | $S \rightarrow E\ T\ E$ | $E \rightarrow c$ |
| $S \rightarrow a\ S$ | $T \rightarrow b\ T$ | $T \rightarrow e$ |
| $S \rightarrow S\ a$ | | |

   The language which is generated by this grammar is regular.

   True                    False

   (b) [5 marks] Consider the TM $M$ as provided by this machine schema:




Assume that TM's are encoded by a string over $\{I, c\}^*$ as described in class. The question: *Does this TM M halt on input "M"?* is Turing-decidable.

True                    False

(c) [5 marks] Let $L = \{ u\, u^R \;\; v\, v^R$ where $u$ and $v$ are strings over $\{a, b\}^*\}$.

Here is a PDA with start state $s$ and final states $\{ f \}$:

| State | Read | Pop | Next | Push |
|---|---|---|---|---|
| s | a | e | s | a |
| s | b | e | s | b |
| s | e | e | t | e |
| t | a | a | t | e |
| t | b | b | t | e |
| t | e | e | u | e |

| State | Read | Pop | Next | Push |
|---|---|---|---|---|
| u | a | e | u | a |
| u | b | e | u | b |
| u | e | e | f | e |
| f | a | a | f | e |
| f | b | b | f | e |

This PDA accepts the language $L$ from above.

True False

(d) [5 marks] Turing-decidable languages are closed under complement.

True False

3. [10 marks] Suppose that an input string $w$ is encoded using I's to encode the symbols and c's as separators. The empty string is encoded as just $c$. The symbol encodings you should use are:

| Symbol | Encoding |
|---|---|
| I | I I I |
| c | I I I I |

As an example, $w = I\ c\ c\ I$ would be encoded as:
$“w” = c\ I^3\ c\ I^4\ c\ I^4\ c\ I^3\ c.$

Prove that $L = \{“w”$ where w is a string over $\{I, c\}^*\}$ is regular by creating a DFA which accepts $L$.

4.(a) [15 marks] Let $L = \{\ c\ I^n\ c\ I^{n+1}\ c\ I^{n+2}\ c\ :\ n \geq 0\ \}$. Use the pumping theorem for context-free languages to prove that $L$ is not context-free.

(b) [5 marks] How can you use your result from (a) and some reasonable assumptions about encodings of Turing machines to conclude that $K = \{\text{“}M\text{”}\ \}$ is not a context-free language?

5. The proof that INDEPENDENT SET is NP-complete describes how to create a graph from a 3-SAT system such that the graph has an independent set of order $n/3$ where $n$ is the number of vertices in the graph if and only if the 3-SAT problem has a satisfying truth assignment.

(a) [5] Apply this procedure for creating the graph to the 3-SAT system: $\{x_1, x_2, x_3\}, \{x_1, \bar{x}_2, \bar{x}_3\}, \{\bar{x}_1, \bar{x}_2, x_4\}, \{x_2, \bar{x}_3, x_4\}$.

(b) [5] Explain why the graph created for part (a) has an independent set of order $n/3$ where $n$ is the number of vertices in the graph if and only if the 3-SAT problem has a satisfying truth assignment.

6. [10 marks] For this question, you may use the fact that Problem *P*: *Given M, does M halt when started on a blank tape?* is not decidable, but do not use any other results. Starting M on a blank tape means starting with (s, # [#]) and starting M on input w means starting with (s, # w [#]), where s is the start state of M.

   Consider Problem *Q*: *Given two Turing machines, is there any string over alphabet* $\Sigma$ *on which they both halt?*
   Prove that Problem *Q* is not decidable.

7.(a) [3 marks] Define $L^*$.

(b) [7 marks] Prove that Turing-decidable languages are closed under Kleene star. You can assume that you have some function in_L($w$) which returns **true** when $w$ is in $L$ and **false** otherwise. Use it to derive pseudocode for a routine in_L*($w$) which returns **true** when $w$ is in $L^*$ and **false** otherwise.

Use this page if you need extra space. Clearly indicate the question you are answering.

**The End.**