Name: ______________________________

ID Number: ______________________________

UNIVERSITY OF VICTORIA

EXAMINATIONS- AUGUST 2002

CSC 320 - Foundations of Computer Science

Instructor: W. Myrvold

Time: 3 hours

TO BE ANSWERED ON THE PAPER.

Instructions:

Students **MUST** count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

This question paper has ten pages (the last page is blank in case you need extra space) plus the header page.

Use only space provided on exam for answering questions. Closed book. No aids permitted.

Question	**Value**	**Mark**
1	20	
2	10	
3	20	
4	20	
5	10	
6	10	
7	10	
Total	**100**	

1. [20] For each of the following languages, indicate the most restrictive of the classes below into which it falls

   (a) finite

   (b) regular

   (c) context-free

   (d) Turing-decidable

   (e) Turing-acceptable

   (f) None of the above.

Example:

$L = \{ a^n b^n : n \geq 0 \}$ The correct answer is (c) since L is context-free, but is not regular.

In this question, we will use $''M''$ to denote the encoding of a Turing Machine M and $''w''$ to denote the encoding of the string w.

_____ i) $L = \{ w\, w : w \in \{ a \}^* \}$

_____ ii) $L = \{ w\, w : w \in \{ a , b \}^* \}$

_____ iii) $\{ w\, w^R : w \in \{a , b\}^* \}$

_____ iv) $L = (a \cup b)^* \phi\, \phi^*$

_____ v) $\{ a^p : p \textit{ is a prime number} \}$

_____ vi) $L = \{ a^n b^m c^p : n \leq p \leq 3n \}$

_____ vii) $L = \{ w \in \{ a, b \}^* : w \textit{ does not contain } aa, ab, ba, \textit{ or } bb \textit{ as a substring} \}$

_____ viii) $L = \{ a^n b^n c^n : n \geq 0 \}$

_____ ix) $\{ \textit{strings in } \{ a , b\}^* \textit{ with an odd number of } a\text{'s} \textit{ and an odd number of } b\text{'s} \}$

Question #1 (continued)

(a) finite

(b) regular

(c) context-free

(d) Turing-decidable

(e) Turing-acceptable

(f) None of the above.

_____ x) { *strings in* $\{ a , b\}^*$ *with length less than 1,000,000* }

_____ xi) $L =$ *the complement of* $\{ a^n b^n c^n : n \geq 0 \}$

_____ xii) $L = \{ u u^R v v^R : u, v \in \{a, b\}^* \}$

_____ xiii) { "M" "w" : *M is a TM, w is an input string* }

_____ xiv) { "M" "w" : *TM M halts on input w* }

_____ xv) { "M" "w" : *TM M does not halt on input w* }

_____ xvi) { "D" "w" : *D is a DFA which accepts w*}

_____ xvii) { "M" "c" : *TM M never prints a "c" when started on a blank tape* }

_____ xviii) { "M" "w" : *TM M halts on input w without using more than 700 tape squares*}

_____ xix) { "M" : *M writes a nonblank symbol when started on a blank tape* }

_____ xx) { "M" : *there is some string on which M halts* }

2. For parts (a), and (b) below, you must choose two DIFFERENT languages from the four given here and you are required to find a regular expression for one of them, and a DFA for the other. Choose carefully to minimize your effort.

The four languages to choose from:

$L_1 = \{w \in \{a, b\}^* :$ the number of a's in w is equal to the number of b's in w $\}$.

$L_2 = \{w \in \{a, b\}^* :$ w contains *aabab* and *babaa*$\}$

$L_3 = \{ w\, w : w \in \{ a , b \}^* \}$

$L_4 = \{w \in \{a, b\}^* :$ the number of a's in w is odd and the number of b's is 3k for some integer k $\}$. Fill in your choices for each part:

Part	Requirement	Language chosen
(a)	Regular Expression	
(b)	Deterministic Finite Automaton	

(a) [5] Give a regular expression for one of the languages.

(b) [5] Give a deterministic finite automaton for the other language that you chose.

3.(a) [10] Let $L_1 = \{ c\, x\, c\, 1^r\, c\, 1^s\, c : x \;\; is \;\; in \;\; \{0,1\}^* \;\; and \;\; x = y\, 1\, z \;\; where \;\; |y| = r - 1 \}$. Give a context-free grammar which generates L_1.

(b) [10] Let $L_2 = \{ c\, x\, c\, 1^r\, c\, 1^s\, c : x \;\; is \;\; in \;\; \{0,1\}^* \;\; and \;\; x = y\, 1\, z \;\; where \;\; |y| = s - 1 \}$. Prove that L_2 is context-free by creating a pushdown automaton which accepts L_2. Design your PDA ‘‘from scratch’’ (without first finding a grammar for the language). For complete marks, use only one symbol, c, in your stack alphabet. Include lots of comments.

START STATE: FINAL STATES:

State	Read	Pop	Next	Push	Comments

4. Let $L = \{ c\, x\, c\, 1^r\, c\, 1^s\, c : x \textit{ is in } \{0,1\}^* \textit{ and } x = u\, 1\, v \textit{ where } |u| = r - 1 \textit{ and } x = y\, 1\, z \textit{ where } |y| = s - 1 \}$.

(a) [5] Let $w = c\ 1\ c\ 1\ c\ 1\ c$. Why can you not use this w with the pumping theorem to prove that L is not context-free?

(b) [5] Let $w = c\ 0^{n-1}\ 1\ 0\ 1\ c\ 1^n\ c\ 1^{n+2}\ c$. Why can you not use this w with the pumping theorem to prove that L is not context-free?

Question 4 continued.

Recall that $L = \{\, c\, x\, c\, 1^r\, c\, 1^s\, c : x \textit{ is in } \{0,1\}^* \textit{ and } x = u\, 1\, v \textit{ where } |u| = r-1 \textit{ and }$ $x = y\, 1\, z \textit{ where } |y| = s-1\}$.

(c) [10] Prove that L is not context-free by applying the pumping theorem to $w = c\ 0^{n-1}\ 1\ c\ 1^n\ c\ 1^n\ c$.

5. Suppose you are given a TM C which makes a copy of an input string w defined over $\Sigma = \{a, b\}$. Starting with # w [#] on the input tape, the final result is # w # w [#]. You are also given a TM M_1 which when started with #w[#] halts with #Y[#] if w is in some language L_1 and #N[#] if w is not in L_1 and you also have such a machine for some other language L_2.

(a) [6] Give the machine schema for a machine which decides L_1 intersect L_2.

(b) [4] What does part (a) tell you about closure under intersection?

6. [10] Assume that $L_{\#} = \{''M'' :\ \textit{TM M halts when started on a blank tape}\}$ is not decidable. Use this fact to prove that $L = \{''M'' :\ \textit{TM M halts on all inputs}\ \}$ is not decidable.

7. The proof that INDEPENDENT SET is NP-complete describes how to create a graph from a 3-SAT system such that the graph has an independent set of order $n/3$ where n is the number of vertices in the graph if and only if the 3-SAT problem has a satisfying truth assignment.

(a) [5] Describe the general procedure for creating the graph, and apply it to the 3-SAT system: $\{x_1, x_2, x_3\}$, $\{x_1, \bar{x}_2, \bar{x}_3\}$, $\{\bar{x}_1, \bar{x}_2, x_4\}$, $\{x_2, \bar{x}_3, x_4\}$.

(b) [5] Prove that the graph created using the general construction you provide for part (a) has an independent set of order $n/3$ where n is the number of vertices in the graph if and only if the 3-SAT problem has a satisfying truth assignment.

Use this page if you need extra space. Clearly indicate the question you are answering.

The End.