CSC 320 Summer 2017: Assignment #3
Due at beginning of class, Fri. June 16

Draw boxes for your marks. Place a 0 in the corresponding box for any questions you omit.

Question	1	2	3	4	5	6	7	8
Marks								

1. Consider the DFA $M = (\{s, t, u\}, \{a, b, c, d\}, \delta, s, \{t\})$ where the transition function δ is given by:

State	Symbol	Next State
s	a	t
s	b	t
s	c	t
s	d	t
t	a	u
t	b	s
t	c	s
t	d	u
u	a	u
u	b	u
u	c	u
u	d	u

(a) [1] Give a regular expression for the language accepted by this DFA.

(b) [1] List all strings of length at most 3 accepted by this DFA.

(c) [1] Give a formula (in terms of k) for the number of strings of length $n = 2k + 1$ accepted by this DFA.

(d) [7] Prove that your formula from (c) is correct by induction.

2.(a) [3] Let $w = a^r b^{2r} c^r$. Describe mathematically all possible ways of choosing x, y, and z such that $w = x\ y\ z$ and y is not the empty string.

(b) [5] Let $L = \{a^r b^s c^t : s \geq 2r, t \leq r^2\}$. Prove that L is not regular using $w = a^r b^{2r} c^r$. Assume only that r is chosen so that $4r \geq k$, where k is the number of states of the DFA.

(c) [2] A more judicious choice of w would simplify your proof from (b). How would you choose such a string w? Which cases are required from part (b) in your revised proof?

3. Let L be a language accepted by a DFA with four states. Assume that the string $bbba \in L$.

(a) [4] Give regular expressions for seven infinite languages $L_1, L_2, \cdots L_7$ such that $L_i \subseteq L$ for at least one value of i.

(b) [2] Prove that L from part (a) is an infinite language.

(c) [4] State what the pumping lemma says about a language accepted by a DFA with four states. What needs to be true in order for you to prove that a language L is NOT accepted by a DFA with four states?

4. Let $L = \{(aab)^r c^s : s \geq r,$ and r, s $\geq 0\}$. Let $w = (a\,a\,b)^n c^n$ for some integers n and k such that $k = 3n$.

(a) [9] Systematically enumerate all ways of factoring w as $w = xyz$ such that $|y| \geq 1$ and $|xy| \leq k$. Number your cases so that you can refer to them easily for parts (b) and (c).

(b) [3] If you use the pumping lemma and this string w to prove that L is not regular, for which cases from (a) can you pump zero times? Show what you get when pumping zero times in each of these cases and explain why the resulting strings are not in L.

(c) [3] For the remaining cases, show what you get when you pump twice and explain why the resulting strings are not in L.

5. The purpose of this question is to prove that the pumping lemma is a necessary but not a sufficient condition for a language to be regular. Consider the language
$L_1 = \{u\, u^R\, v \ : \ u, v \ \in \ \{0, 1\}^+\}$
(u and v are strings made up of 0's and 1's of length at least one).

(a) [2] How many strings are in in L_1 with length at most 6? Justify your answer by telling me the choices for a minimal prefix u and the number of choices left for v in each case.

(b) [4] The purpose of part (b) is to double check your answer to part (a) and to make sure you understand the language L. Write a program which counts the number of strings in L for each length k for $k \leq 16$. To check the strings of length k, you can use a loop like this:

```
for (i=0; i < (1 << k); i++)
{
    1. Create a string w which corresponds to the
       binary representation of the integer i.
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2. Check if w is in L.

}

Hand in your program and its output on paper.

(c) [3] Let $L_2 = L_1 \cap (01)^* 11(10)^*$ Describe L_2 using a set descriptor notation. Don't forget about v.

(d) [10] Prove that L_2 is not regular using the pumping lemma.

(e) [2] Given that that L_2 is not regular use this to prove that L_1 is not regular.

(f) [4] For all strings w in the language L_1 such that the length of w is at least four, prove that there exists some way to factor w as $w = x\,y\,z$ such that $|xy|$ is at most four, y is not the empty string and xy^*z is a subset of L_1. Hint: Break it into two cases (one where the length of u is one and the other case where the length of u is greater than one).

6. Regular context-free grammars.

(a) [7] Use a DFA for the language $L = \{w \in \{a, b\}^*$: the number of a's is odd and the number of b's is not divisible by 3$\}$ to construct a regular context-free grammar for L.

(b) [3] Give a context-free grammar for L from part (a) which is not regular.

7. Design context-free grammars for the following languages:

(a) [3] $L_1 = \{u\,u^R\,v\,v^R : u, v \in \{0, 1\}^*, |u| \geq 2, |v| \geq 1\}$.

(b) [3] $L_2 = \{1^p\,0^q\,1^r\,0^{3q}\,1^s : p \geq 2, q \geq 0\ , r \geq 0\ s \geq 3\}$.

(c) [3] $L_3 = \{a^n b^m : 2\,n \leq m \leq 5\,n\}$.

(d) [3] $L_4 = a^p\,b^q\,c^r\,d^s$ where $(r + s) = 2\,(p + q)$, and $p, q, r, s \geq 0\}$.

8. [8] Our definition of a regular express over the alphabet $\{a, b\}$ is that

[Basis] ϕ, a, and b are regular expressions.

[Inductive step] If α and β are regular expressions then so are

(i) $(\alpha \cup \beta)$

(ii) $(\alpha\beta)$

(iii) α^*

Construct a context-free grammar that generates all regular expressions over $\{a, b\}$.

Important: use this definition for a regular expression instead of considering regular expressions that have the (and) symbols omitted when not needed for precedence.