# Problem of the Day

1. Step through the code on the following page drawing pictures of what happens if the input list represents the big integer 999.

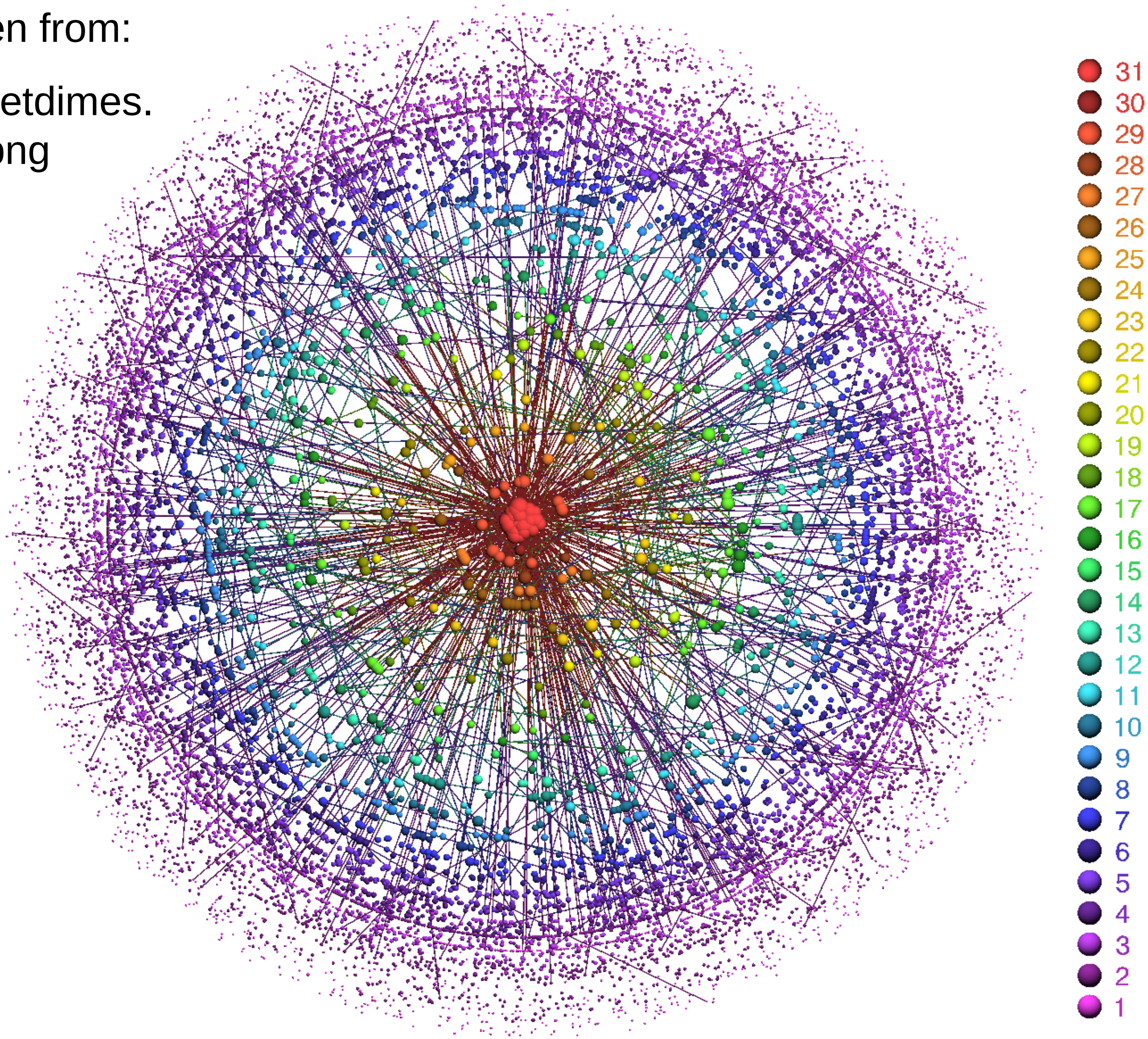
2. How much space does this recursive routine use in addition to the space used by ListNode's?

```
public void plus_plus()
{
  start.data++;
  if (start.data != 10)
      return;
  start.data = 0;
  if (n==1)
  {
    rear.next = new
          ListNode(1, null);
    rear= rear.next;
    n++;
    return;
  }
```

```
// CONTINUED,  n > 1

  LinkedList list1 =  new
       LinkedList(n-1,
       start.next, rear);

  list1.plus_plus();

  rear= list1.rear;

  n= list1.n + 1;
}
```

Internet taken from:

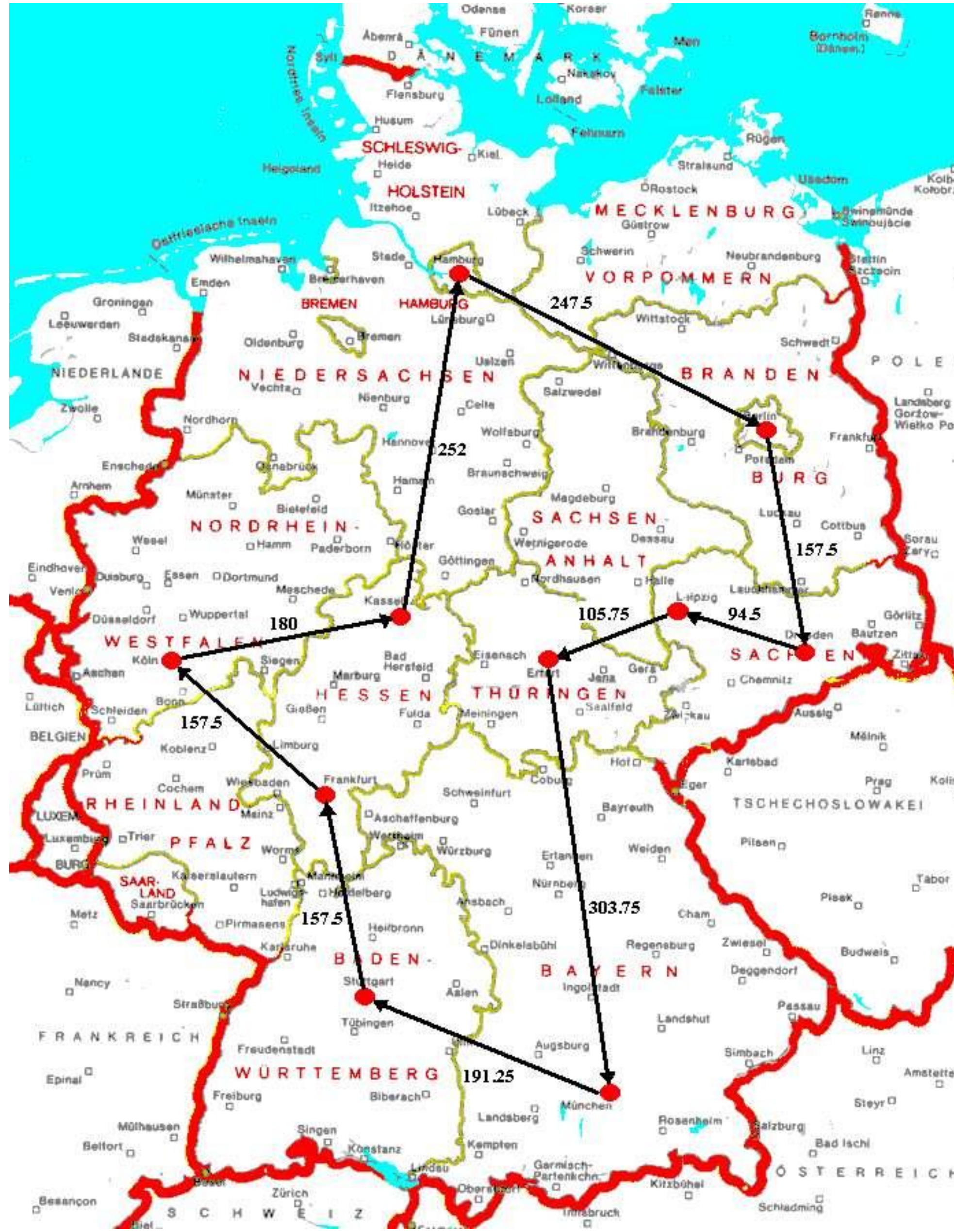
http://www.netdimes.org/asmap.png

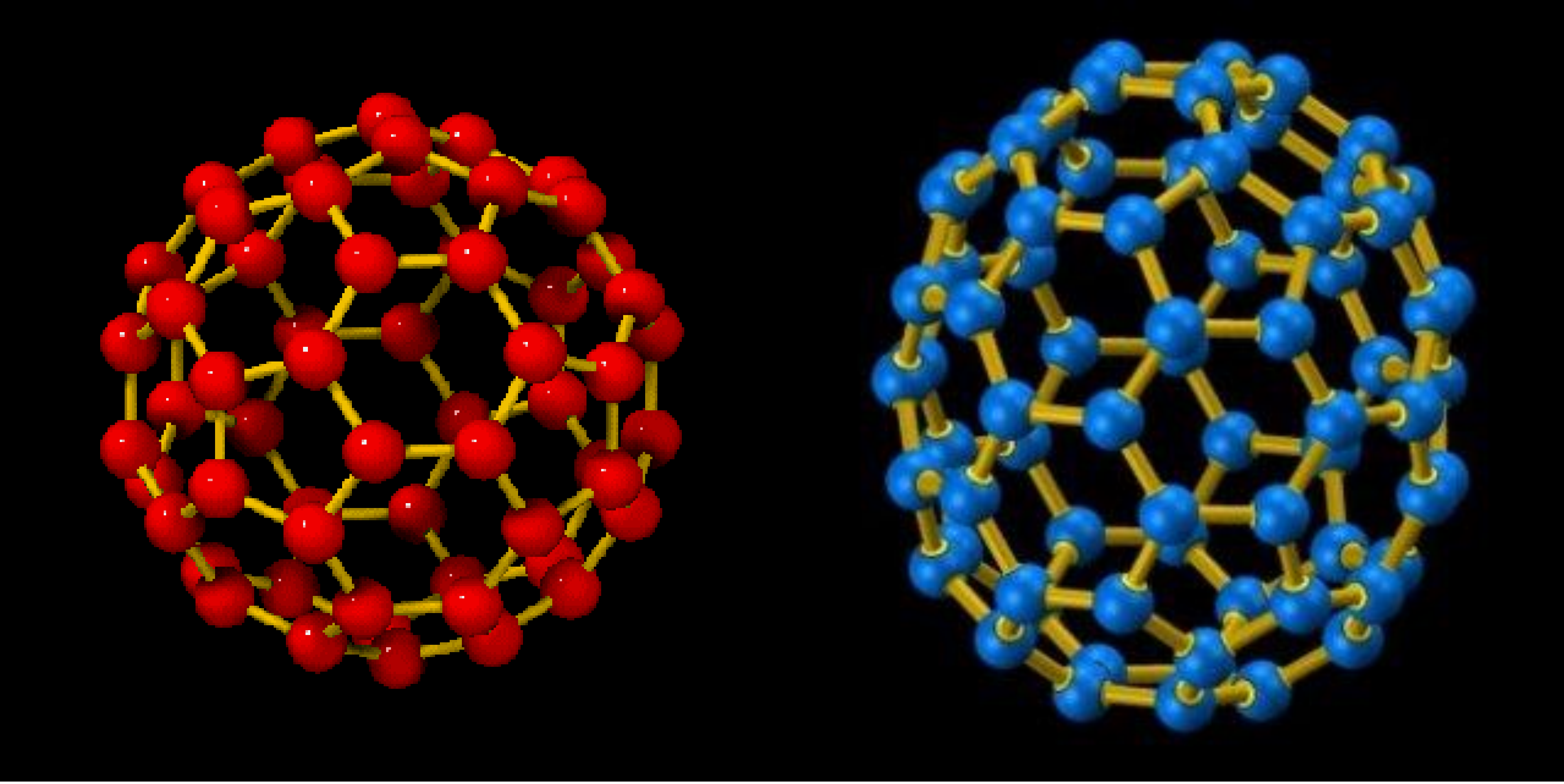


Travelling Salesman

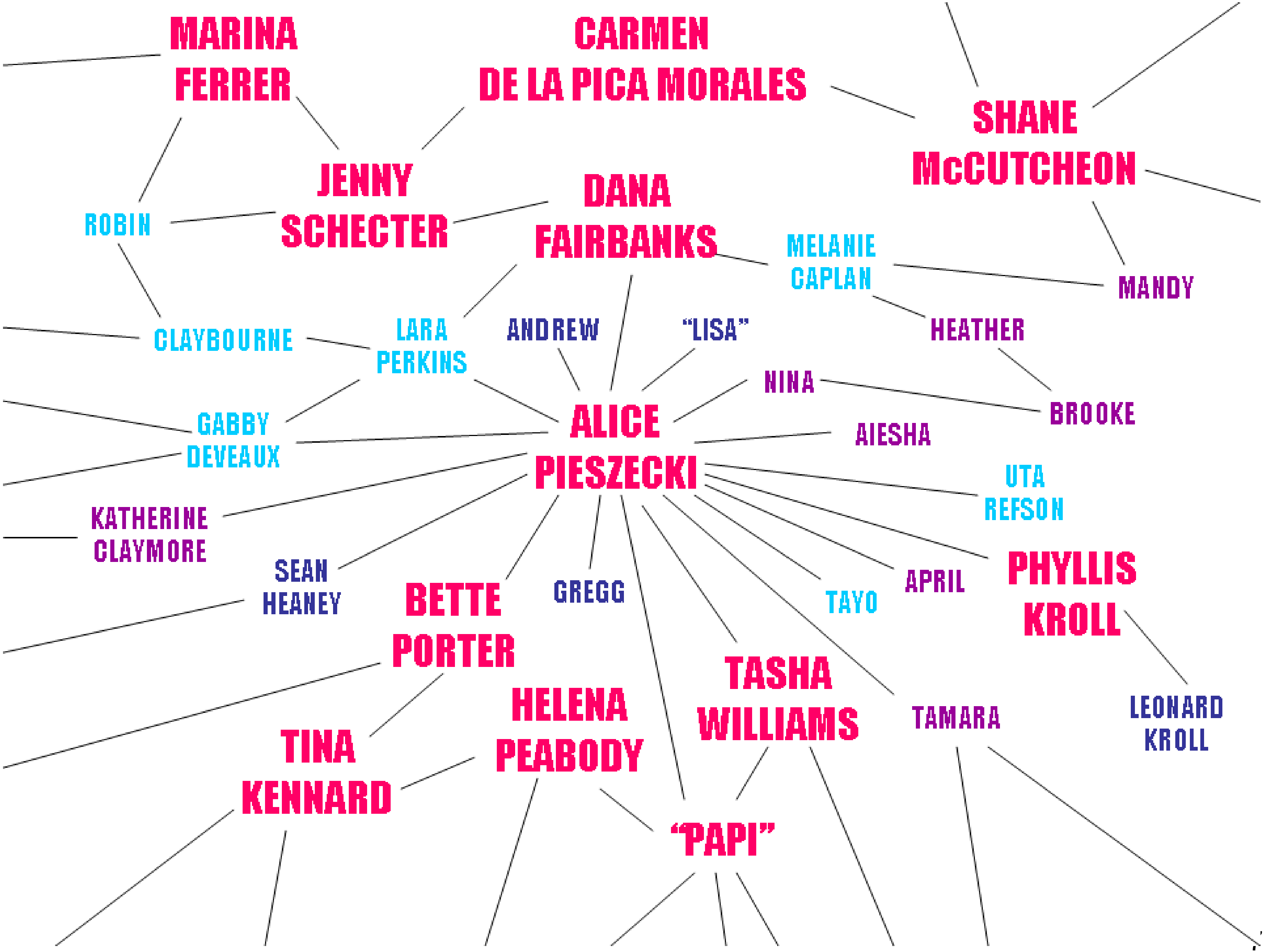
From:

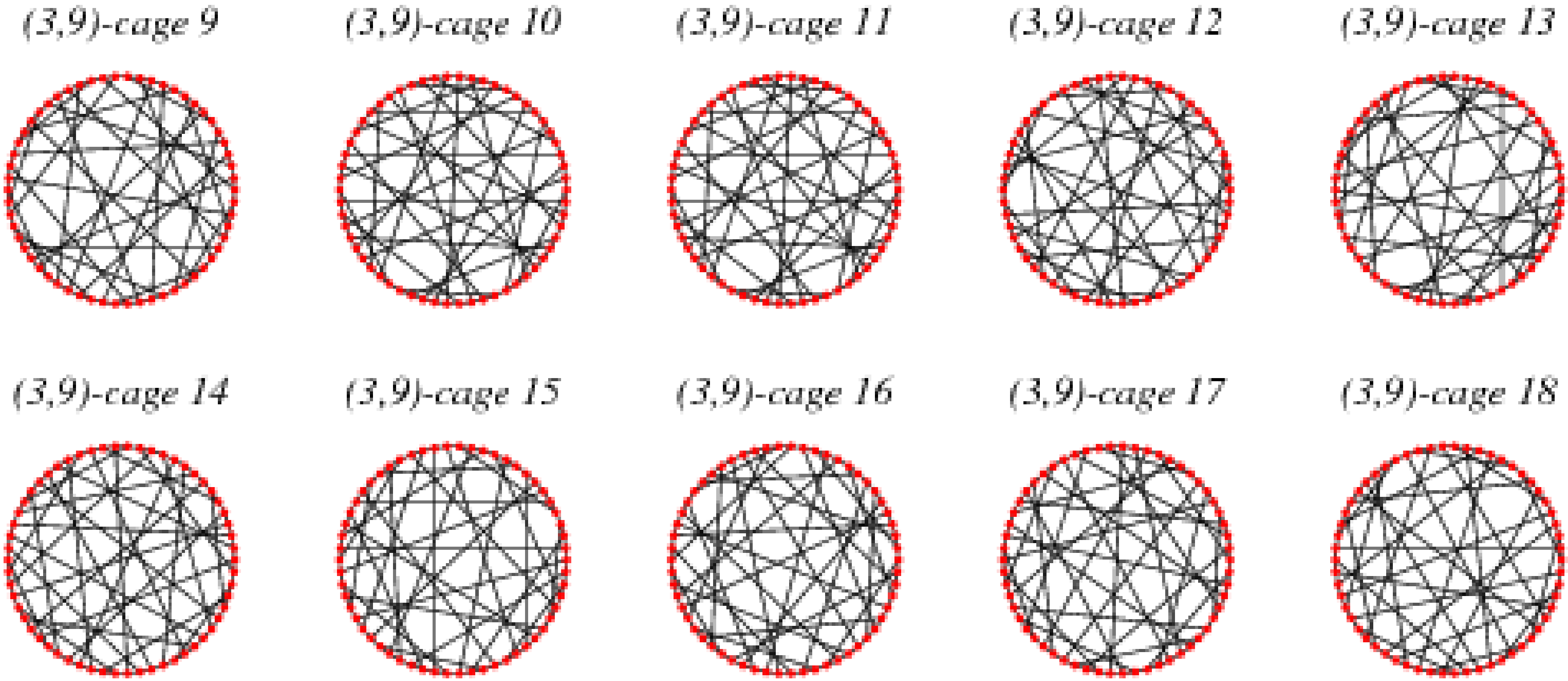
Ehsan Moeinzadeh
Guildford, Surrey,
United Kingdom





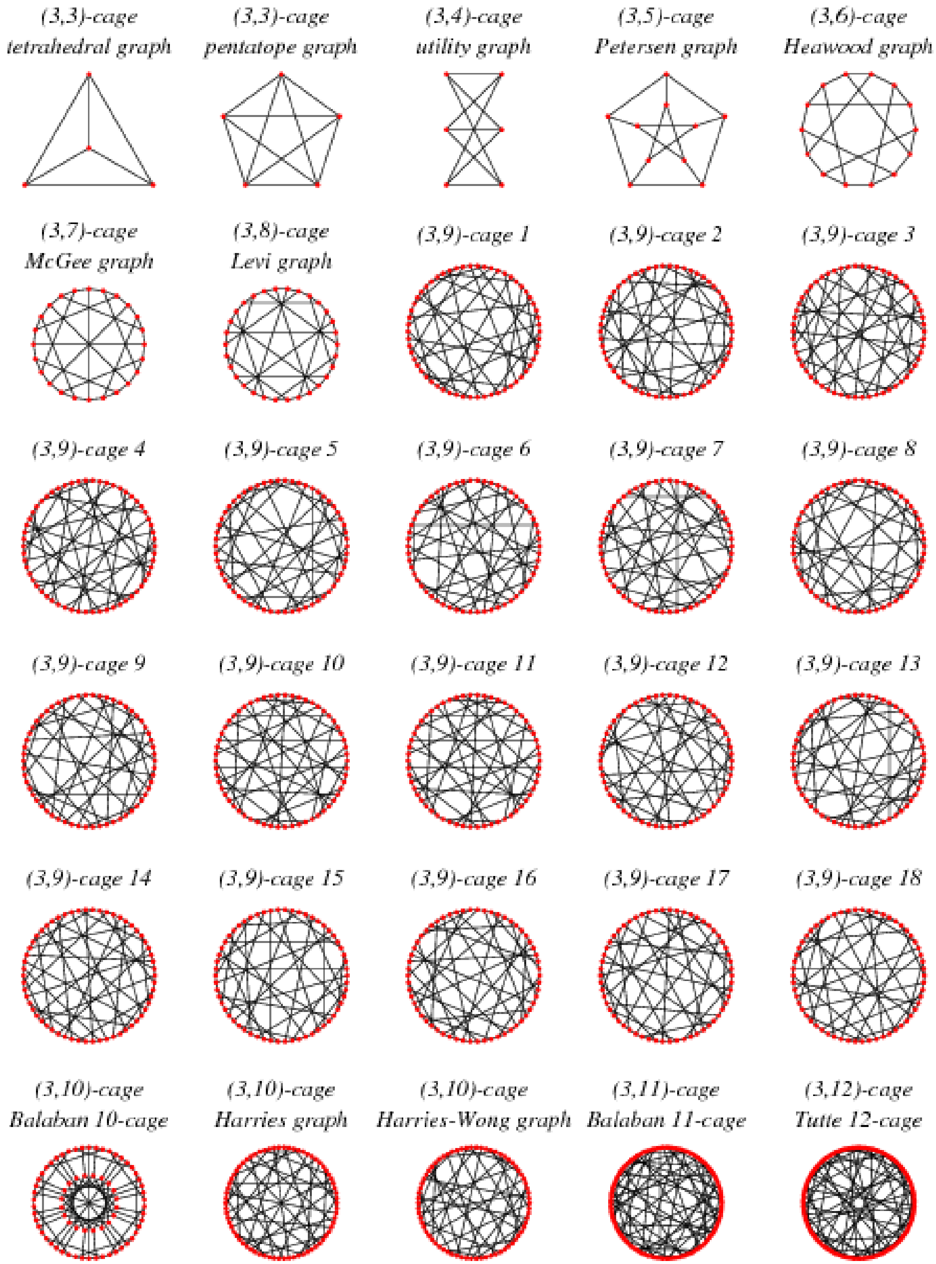
Graphs representing chemical molecules


MARINA FERRER
CARMEN DE LA PICA MORALES
SHANE McCUTCHEON
JENNY SCHECTER
DANA FAIRBANKS
ROBIN
MELANIE CAPLAN
MANDY
CLAYBOURNE
LARA PERKINS
ANDREW
"LISA"
HEATHER
NINA
BROOKE
GABBY DEVEAUX
ALICE PIESZECKI
AIESHA
UTA REFSON
KATHERINE CLAYMORE
SEAN HEANEY
GREGG
APRIL
TAYO
PHYLLIS KROLL
BETTE PORTER
TASHA WILLIAMS
HELENA PEABODY
TINA KENNARD
TAMARA
LEONARD KROLL
"PAPI"




Pictures from:

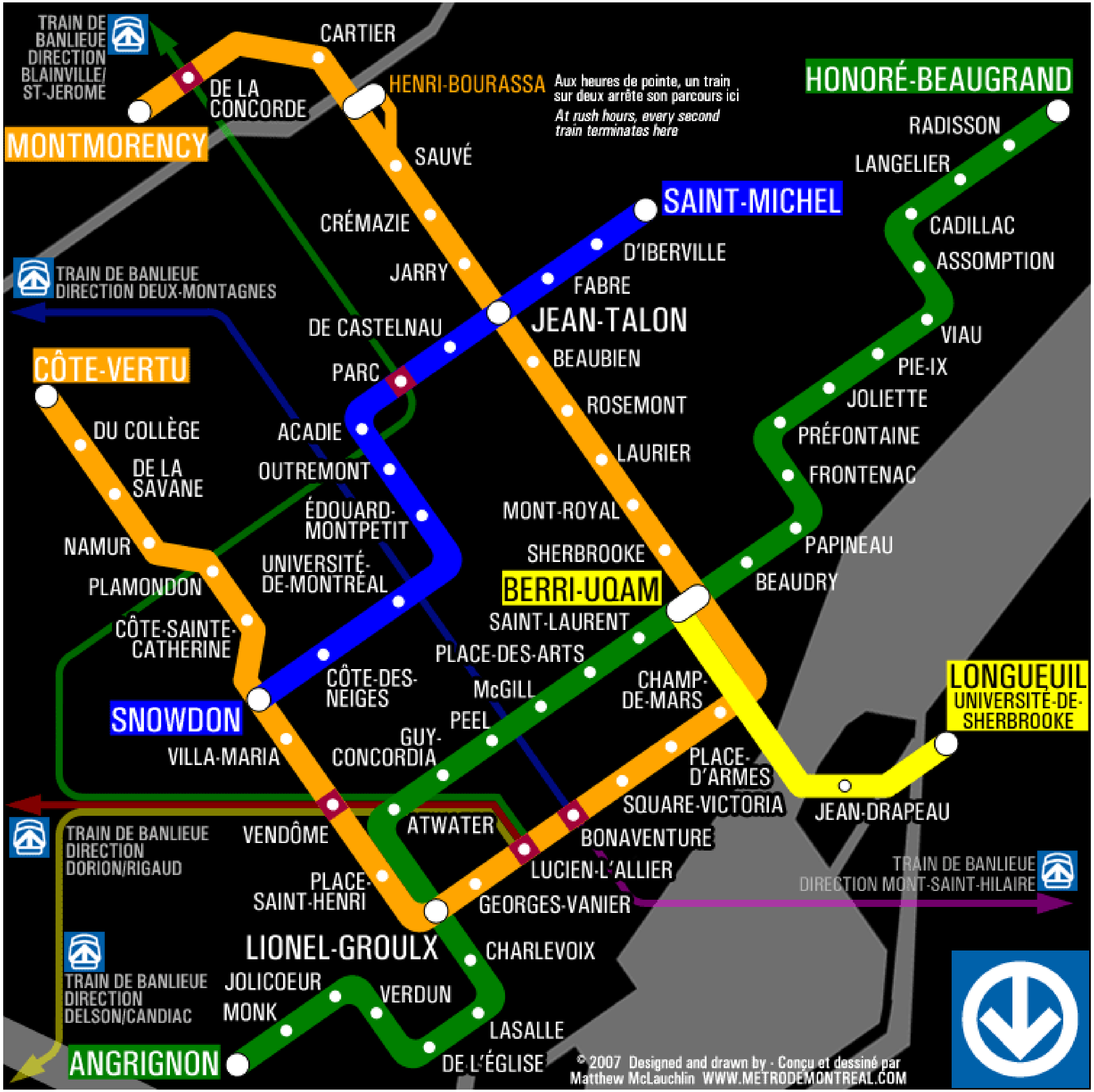
http://mathworld.wolfram.com/CageGraph.html




# Cages: co-starred on The Big Bang Theory

Pictures from:

http://mathworld.wolfram.com/CageGraph.html


TRAIN DE BANLIEUE DIRECTION BLAINVILLE/ ST-JEROME
MONTMORENCY
DE LA CONCORDE
CARTIER
HENRI-BOURASSA
Aux heures de pointe, un train sur deux arrête son parcours ici
At rush hours, every second train terminates here
HONORÉ-BEAUGRAND
RADISSON
LANGELIER
CADILLAC
ASSOMPTION
SAUVÉ
CRÉMAZIE
JARRY
SAINT-MICHEL
D'IBERVILLE
FABRE
JEAN-TALON
TRAIN DE BANLIEUE DIRECTION DEUX-MONTAGNES
DE CASTELNAU
PARC
BEAUBIEN
ROSEMONT
LAURIER
VIAU
PIE-IX
JOLIETTE
PRÉFONTAINE
FRONTENAC
CÔTE-VERTU
DU COLLÈGE
DE LA SAVANE
ACADIE
OUTREMONT
ÉDOUARD-MONTPETIT
UNIVERSITÉ-DE-MONTRÉAL
MONT-ROYAL
SHERBROOKE
BERRI-UQAM
PAPINEAU
BEAUDRY
NAMUR
PLAMONDON
CÔTE-SAINTE-CATHERINE
SAINT-LAURENT
PLACE-DES-ARTS
CÔTE-DES-NEIGES
McGILL
CHAMP-DE-MARS
LONGUEUIL UNIVERSITÉ-DE-SHERBROOKE
SNOWDON
VILLA-MARIA
GUY-CONCORDIA
PEEL
PLACE-D'ARMES
JEAN-DRAPEAU
SQUARE-VICTORIA
TRAIN DE BANLIEUE DIRECTION DORION/RIGAUD
VENDÔME
ATWATER
BONAVENTURE
LUCIEN-L'ALLIER
TRAIN DE BANLIEUE DIRECTION MONT-SAINT-HILAIRE
PLACE-SAINT-HENRI
GEORGES-VANIER
LIONEL-GROULX
CHARLEVOIX
TRAIN DE BANLIEUE DIRECTION DELSON/CANDIAC
JOLICOEUR
MONK
VERDUN
LASALLE
ANGRIGNON
DE L'ÉGLISE
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# Phylogenetic trees

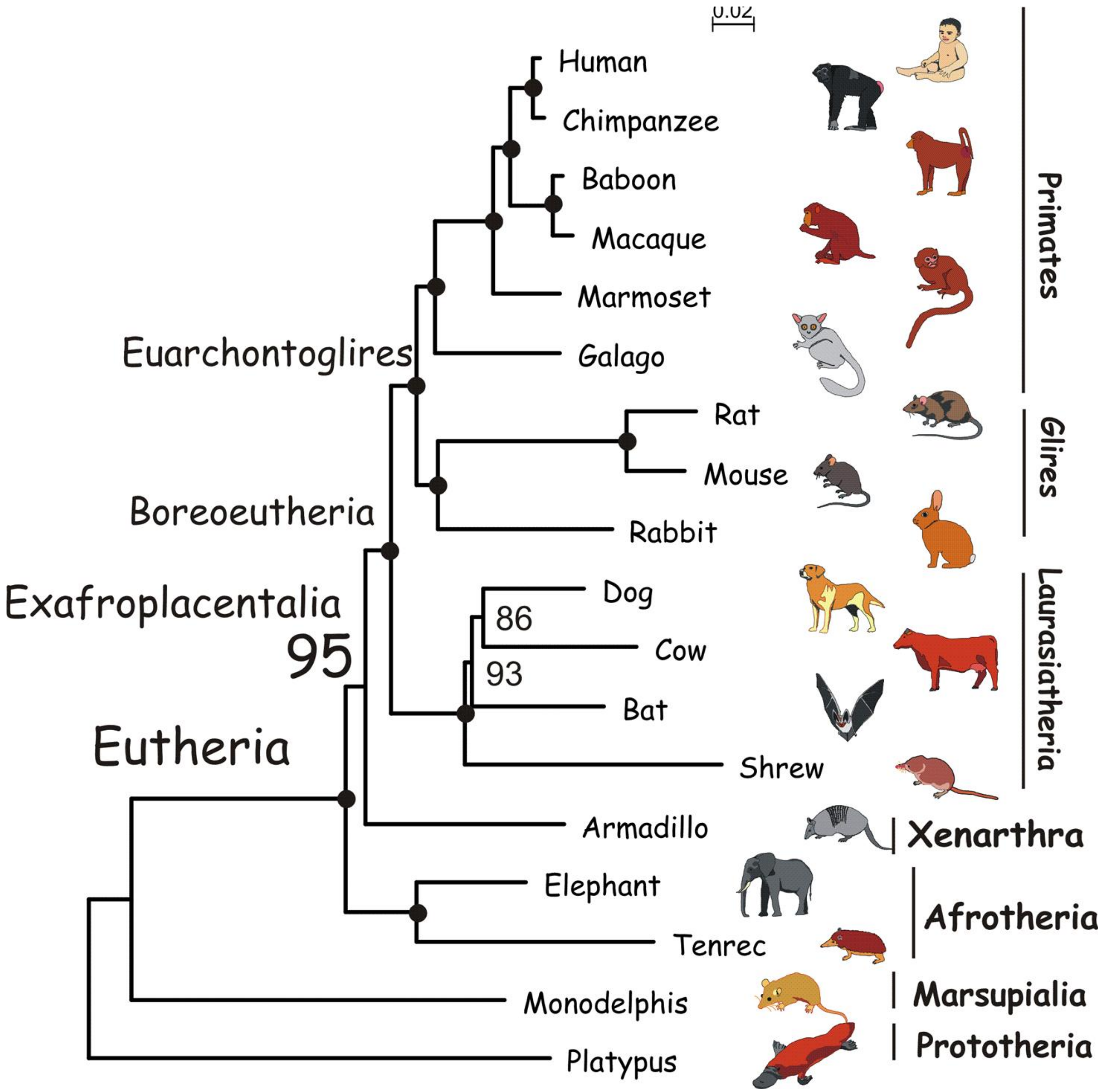



Image from:
http://journals.plos.org/plosgenetics/article?id=10.1371/journal.pgen.0030002

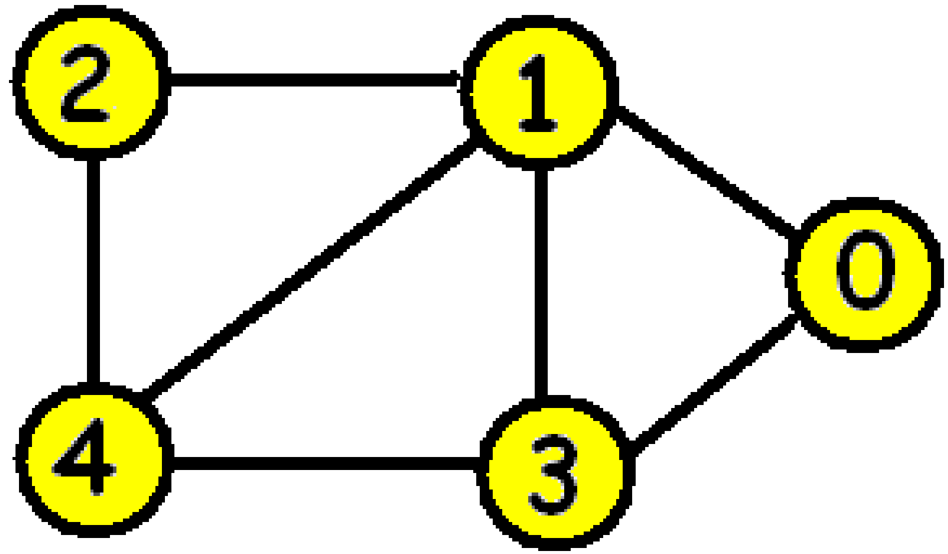
## Some graph theory applications:

- Transportation networks
- Computer networks
- Phone or cable networks
- Dependencies between different methods in a computer program
- Social networks

# Data Structures for Graphs

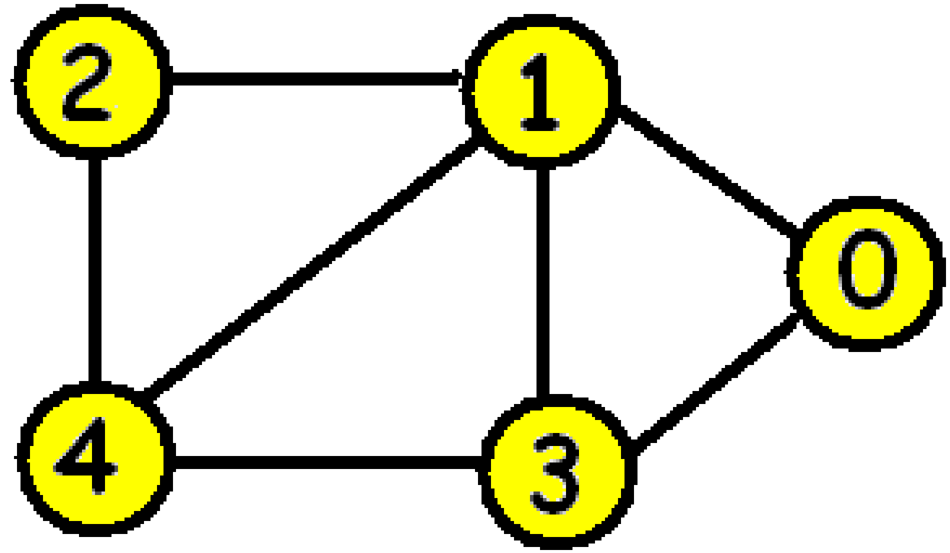
How can graphs be stored in the computer?

How does this affect the time complexity of algorithms for graphs?

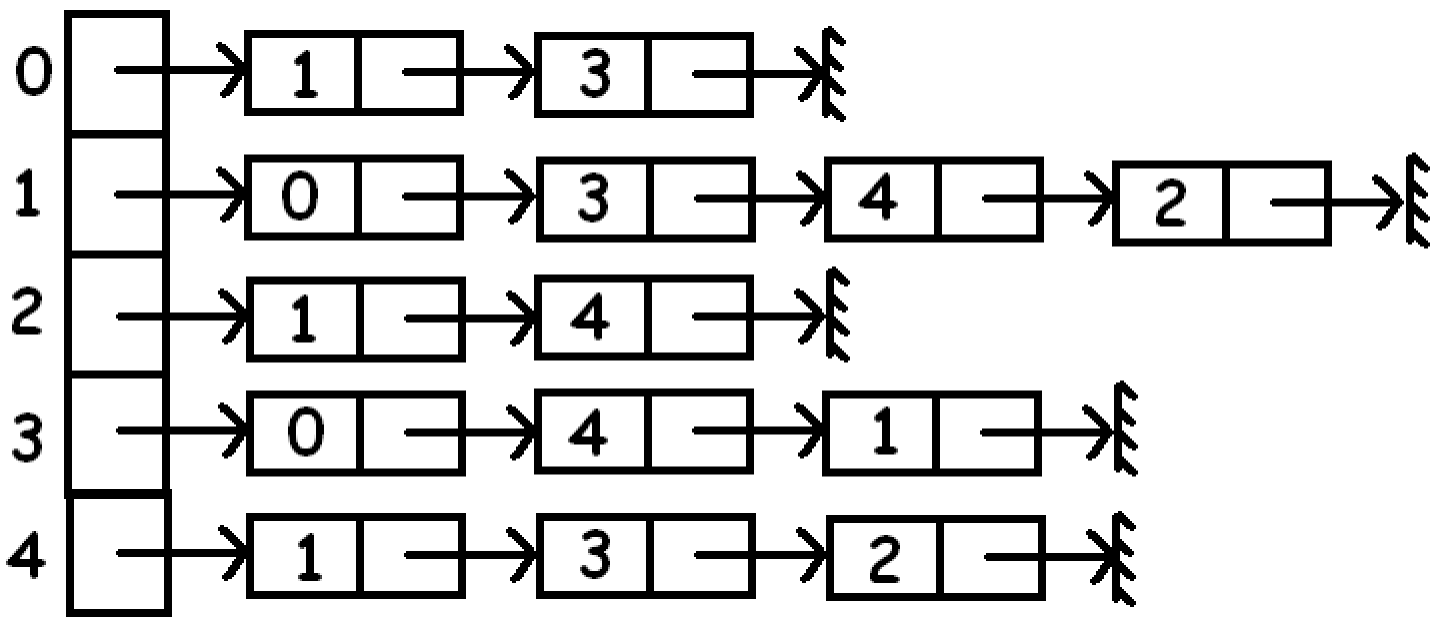



Adjacency matrix:

| | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 | 1 |
| 3 | 1 | 1 | 0 | 0 | 1 |
| 4 | 0 | 1 | 1 | 1 | 0 |




## Adjacency list:



# Adjacency lists:

Lists can be stored:

1. sorted,
2. in arbitrary order,
3. in some other specific order- for example a rotation system has the neighbours of each vertex listed in clockwise order in some planar embedding of a graph (a picture drawn on the plane with no edges crossing).

## Data structures for graphs:

n= number of vertices

m= number of edges

Adjacency matrix: Space $\Theta(n^2)$

Adjacency list: Space $\Theta(n + m)$

How long does it take to do these operations:

1. Insert an edge?
2. Delete an edge?
3. Determine if an edge is present?
4. Traverse all the edges of a graph?

A cycle of a graph is an alternating sequence of vertices and edges of the form $v_0$, $(v_0, v_1)$, $v_1$, $(v_1, v_2)$, $v_2$, $(v_2, v_3)$, ... ,$v_{k-1}$, $(v_{k-1}, v_k)$, $v_k$ where except for $v_0 = v_k$ the vertices are distinct.

Exercise: define path, define connected.

A tree is a connected graph with no cycles.

A subgraph H of a graph G is a graph with $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

H is spanning if $V(H) = V(G)$.

Spanning tree- spanning subgraph which is a tree.

# Strange Algorithms

Input: a graph G
Question: does G have a spanning tree?

This can be answered by computing a determinant of a matrix and checking to see if it is zero or not.

For lower bound arguments, it is essential to not make too many assumptions about how an algorithm can solve a problem.