Replace these recurrence relations with simpler ones that would give the same time complexity asymptotically.

For all of these assume $T(1)=1$.

(a) $T(n)= 3 n^2 + 4n - 7 + T(n/2)$

(b) $T(n) = 2 n + 10 \log_2(n) + 100 + 2 T (n/2)$

(c) $T(n) = n – 5 + T(1) + T (n-1)$

(d) $T(n) = 23 + T(n-1) + 2 T(n/2)$

(e) $T(n) = 7 * 2^n + 7 * n^2 + T(n-1)$

Space Requirements

For very big problems, it is sometimes necessary to choose algorithms which do not require too much extra space to avoid crashing the computer.

Algorithms using less space can often run faster than those requiring more space. This is very important for interactive systems such as video games.

We will learn to analyze extra space requirements using the same notation we are using for the running times (Big Oh, Ω and Θ).

Basics

NIST= National Institute of Standards and Technology

Bit. 0 or 1.

Byte. 8 bits.

Megabyte (MB). 1 million or 2^{20} bytes.

Gigabyte (GB). 1 billion or 2^{30} bytes.

NIST ↓ (1 million / 1 billion)

most computer scientists ↓ (2^{20} / 2^{30})

64-bit machine. We assume a 64-bit machine with 8-byte pointers.

- Can address more memory.
- Pointers use more space.

some JVMs "compress" ordinary object pointers to 4 bytes to avoid this cost

Gray background slides are taken with permission from Sedgewick and Wayne.

Typical memory usage for primitive types and arrays

type	bytes
`boolean`	1
`byte`	1
`char`	2
`int`	4
`float`	4
`long`	8
`double`	8

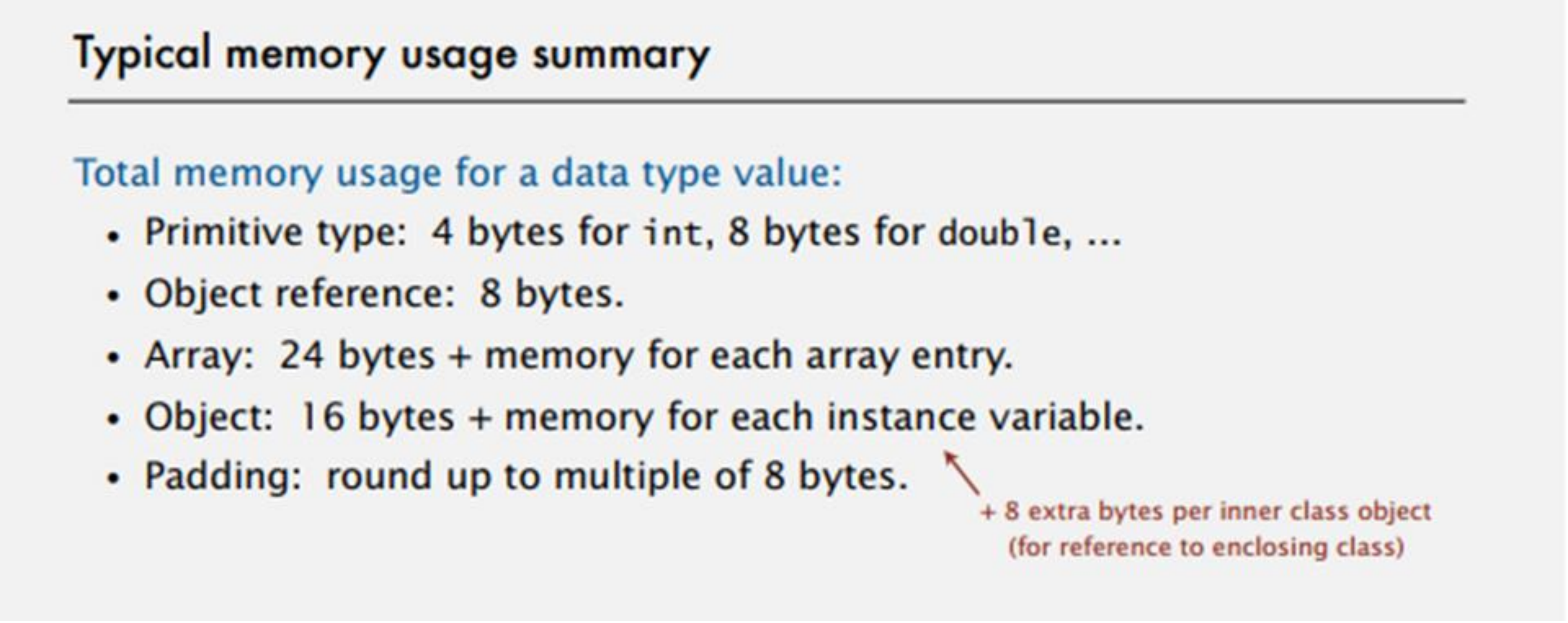
primitive types

type	bytes
`char[]`	$2N + 24$
`int[]`	$4N + 24$
`double[]`	$8N + 24$

one-dimensional arrays

type	bytes
`char[][]`	$\sim 2MN$
`int[][]`	$\sim 4MN$
`double[][]`	$\sim 8MN$

two-dimensional arrays



Typical memory usage summary

Total memory usage for a data type value:

- Primitive type: 4 bytes for `int`, 8 bytes for `double`, ...
- Object reference: 8 bytes.
- Array: 24 bytes + memory for each array entry.
- Object: 16 bytes + memory for each instance variable.
- Padding: round up to multiple of 8 bytes.

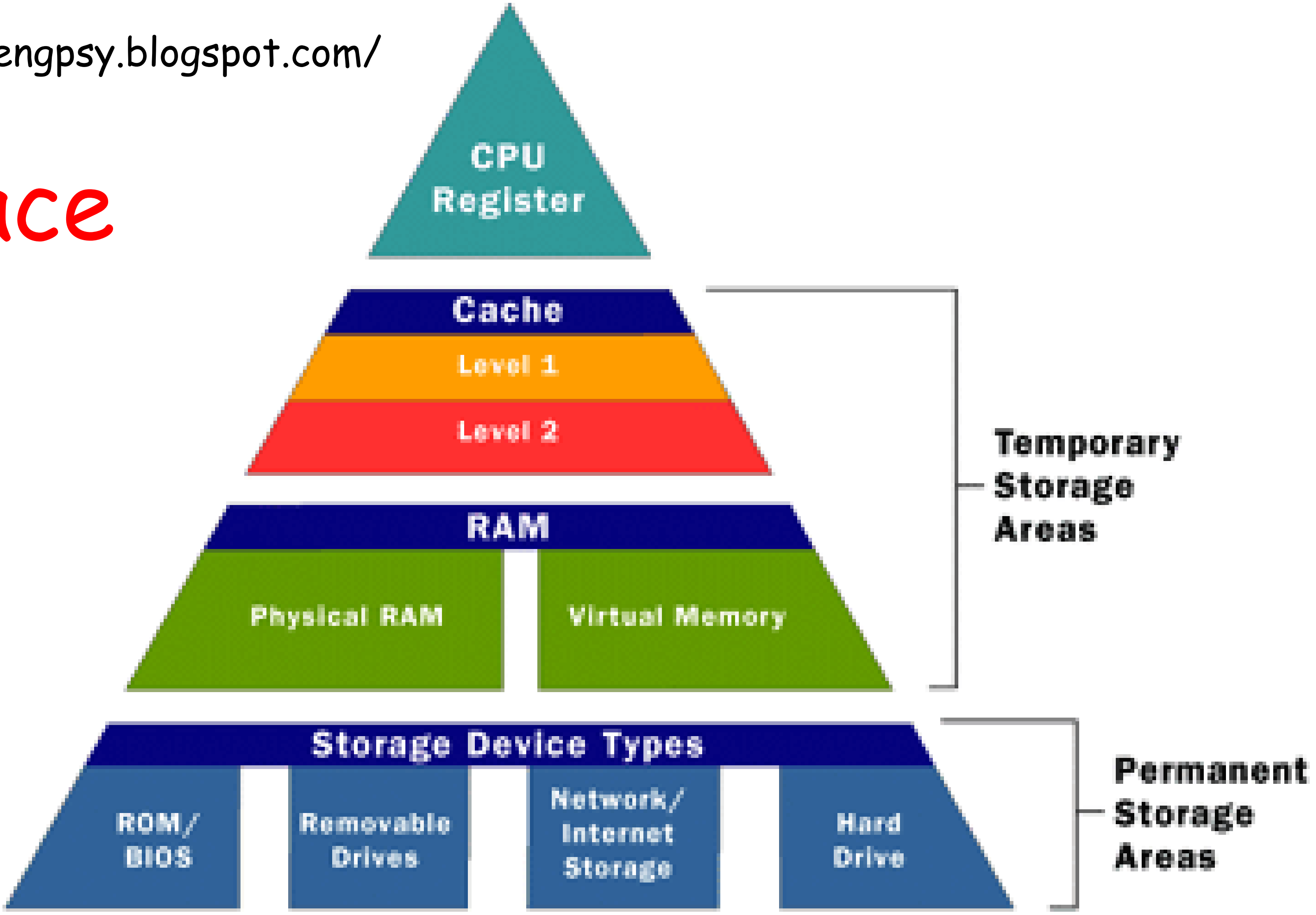
+ 8 extra bytes per inner class object
(for reference to enclosing class)

Objects header info (16 bytes): reference to the class garbage collection info, synchronization info.

Array header info (24 bytes) contains:
16 bytes : object header info
4 bytes: length
4 bytes: padding

http://engpsy.blogspot.com/

Space



L1 cache - 10 nanoseconds, 4-16 kilobytes
L2 cache - 20-30 nanoseconds, 128-512K
Main memory - 60 nanoseconds, 32 MB or more

Extra Space:

For sorting algorithms:

We will only count the space used for the bookkeeping that the algorithm does and not the space that the data is stored in.

Array A with the data: not counted.

Linked list: extra space is used by next pointers.

In the worst case, how much extra space might the program need at one moment in time?

```
 public class Array
 {  int n;  int [] A;
// Space for data in A is not counted
   public void maxSort()
   {    int i, t, end, max_pos;

        for (end= n-1; end > 0; end--)
        {    max_pos=0;
             for (i= 1; i <= end; i++)
                  if (A[i] >= A[max_pos])  max_pos= i;
             t= A[max_pos];
             A[max_pos]= A[end];
             A[end]= t;
         }
 }
```

Implicit variable: this

Recursive MaxSort:

```
public void maxSort(int size)
{     int i, t, maxPos;

      if (size <= 1) return;
      maxPos=0;
      for (i=1; i < size; i++)
            if (A[i] >= A[maxPos]) maxPos=i;
      t= A[maxPos];
      A[maxPos]= A[size-1];
      A[size-1] = t;
      maxSort(size-1);
  }
```

Implicit variable: this

The extra space used is:

Iterative MaxSort: $\theta(1)$

Recursive MaxSort: $\theta(n)$

If space is an issue, iterative MaxSort is a better choice.

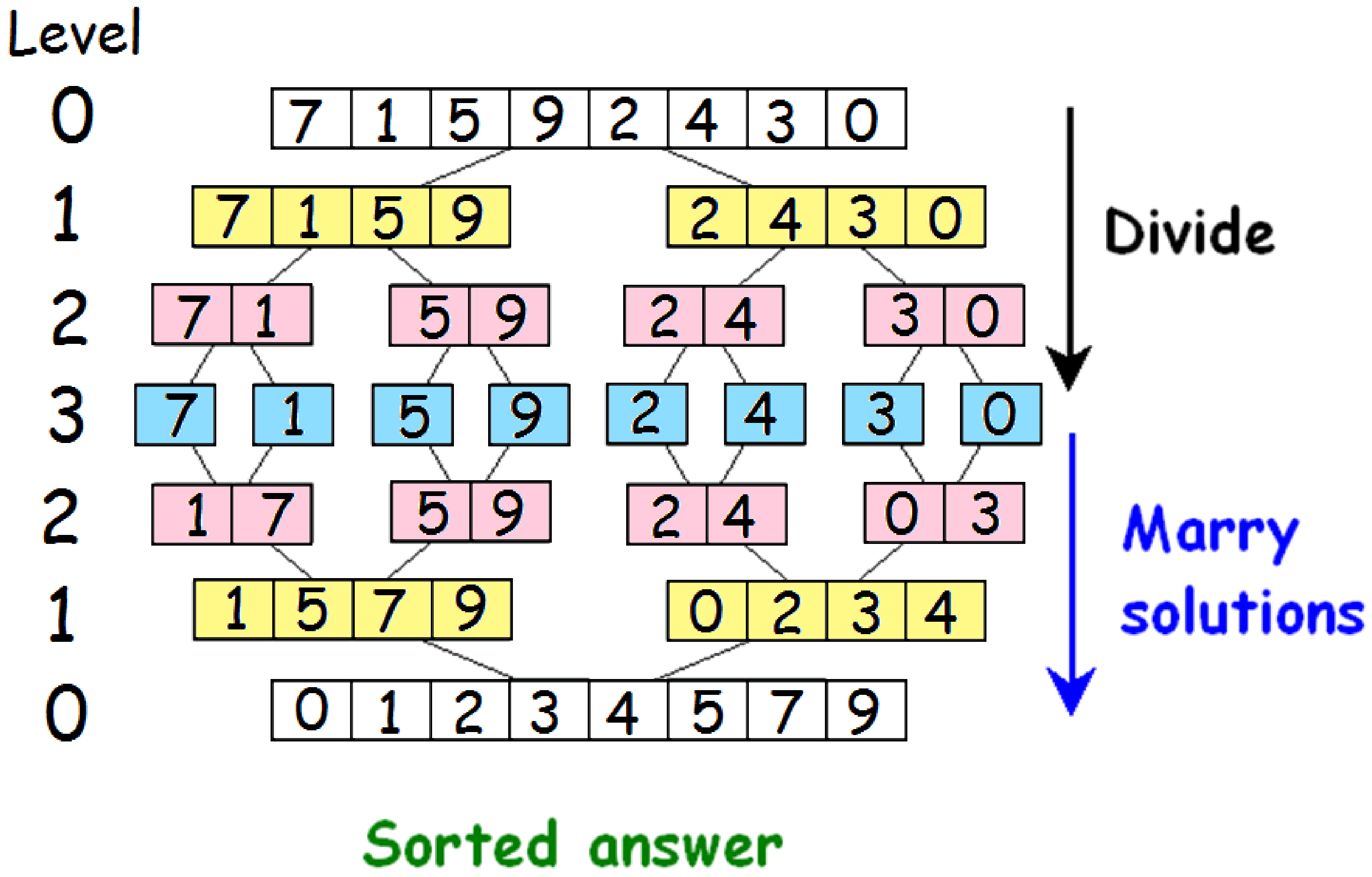
How much extra space does Quicksort use (the implementation presented in class)?

```
// quicksort A[left] to A[right]

public static void quicksort(
                    int[] A, int left, int right)
{
      if (right <= left) return;

      int pivot_pos = partition(A, left, right);

      quicksort(A, left, pivot_pos-1);

      quicksort(A, pivot_pos+1, right);
   }
```

```
// partition A[left] to A[right]
private static int partition(
                int [] A, int left, int right )
{
int i = left; int j = right-1;

while (true)
{
    while (A[i]  < A[right]) {i++;}
    while (A[right] < A[j]) {j--; if (j == left) break; }
    if (i >= j) break;
    swap(A, i, j); i++; j--;
}
swap(A, i, right); // Put pivot element into place
return i;
}
```

How much extra space does Mergesort use if the data structure is a linked list?

Mergesort



Level
0
1
2
3
2
1
0
7 1 5 9 2 4 3 0
7 1 5 9
2 4 3 0
7 1
5 9
2 4
3 0
7
1
5
9
2
4
3
0
1 7
5 9
2 4
0 3
1 5 7 9
0 2 3 4
0 1 2 3 4 5 7 9
Divide
Marry solutions
Sorted answer

```
public class LinkedList // Same as Lab 1
{  int n;
   ListNode start;
   ListNode rear;

   public void mergeSort(int level)
   {
      LinkedList list1;
      LinkedList list2;
      int i;

/*    A list of size 0 or 1 is already sorted. */

      if (n <=1) return;
```

Implicit variable this.

Code omitted here.

I am assuming for this program, the list is split into two sublists list1 and list2 the same way you do it for your reverse method. The list1 has the first floor(n/2) items and the list2 has the next ceiling(n/2).

Make sure that in your code:

list1.start, list1.rear, list1.n, and

list2.start, list2.rear, list2.n all have correct values and that both list1 and list2 are null terminated.

```
/*    Sort the 2 sublists recursively: */

      list1.mergeSort(level+1);

      list2.mergeSort(level+1);
```

```
/*    Merge the two sorted sublists. */
start= null; rear= null;
LinkedList tmp; // Keeps track of list with smallest key
while (list1.start != null && list2.start != null)
{
       if (list1.start.data < list2.start.data)
              tmp= list1;
       else tmp= list2;

       if (start == null) start= tmp.start;
       else rear.next= tmp.start;

        rear= tmp.start;

      tmp.start= tmp.start.next;
      rear.next= null;
}
```

```
//    Now append the list that still has
//    items in it to the end.

      if (list1.start != null)
            tmp= list1;
       else
            tmp = list2;
       rear.next= tmp.start;
// Make sure our
// object has a correct rear pointer
       rear= tmp.rear;

} // end mergeSort
```