1. Let a, b, and d be constants with a, b, d ≥ 0. Prove that

$$2 + 3a\, n + 4\, b\, n \log_2 n + d\, n^2 \in O(n^2)$$

2. Solve this recurrence relation by repeated substitution:

T(k)= k + T(k-1), T(0)= 2.

Start numbering your steps with Step 0:

## Announcements:

Assignment 1B is due on Thursday at the beginning of class.

Some practice problems for Lab 3 have been posted.

Make sure that when you upload programs that they compile!

Consider the following code fragment:

```
sum=2;
for (i=1; i <= n; i++)
{
    sum= sum + i;
}
```

Let T(k) be the value of the variable sum after k iterations of the loop.

1. Set up a recurrence for T(k).
2. Solve it.
3. Prove your answer is correct by induction.

```
sum=2;
for (i=1; i <= n; i++)
{
    sum= sum + i;
}
```

Theorem: After k iterations of the loop, the value of sum is equal to 2+ [k(k+1)/2].

```
sum=2;
for (i=1; i <= n; i++)
{
    sum= sum + i;
}
```

Theorem: After k iterations of the loop, the value of sum is equal to 2+ k(k+1)/2.

[Base case, k=0] The case where k=0 corresponds to the case where the loop has executed 0 times. In this case, sum has its initial value of 2. The formula says that the value of sum is 2 + k(k+1)/2=

2 + 0 * (0+1)/2 = 2 as required.

```
sum=2;
for (i=1; i <= n; i++)
{
    sum= sum + i;
}
```

Theorem: After k iterations of the loop, the value of sum is equal to 2+ k(k+1)/2.

[Induction step]

Assume that after k iterations of the loop, the value of sum is equal to 2+ k(k+1)/2.

We want to prove that after k+1 iterations of the loop, the value of sum is equal to

$2+ (k+1)((k+1)+1)/2= (k^2 + 3k + 6)/2.$

```
sum=2;
for (i=1; i <= n; i++)
{
    sum= sum + i;
}
```

Theorem: After k iterations of the loop, the value of sum is equal to 2+ k(k+1)/2.

Consider iteration k+1 of the loop. The value of i is equal to k+1. By induction, the value of sum just before i is added to it is equal to 2+ k(k+1)/2.

Then i=k+1 is added to sum to give the new value of sum equal to 2+ k(k+1)/2 + (k+1). Simplifying algebraically gives sum = $(4 + k^2 + k + 2k + 2)/2$ and therefore, sum= $(k^2 + 3k + 6)/2$ as required.

```
sum=2;
for (i=1; i <= n; i++)
{
    sum= sum + i;
}
```

Theorem: After k iterations of the loop, the value of sum is equal to 2+ k(k+1)/2.

CONCLUSION:

When the loop finishes executing, the value of sum is equal to:

2+ n(n+1)/2.

Consider the following code fragment:

```
sum=2;
for (i=1; i <= n; i++)
{
    sum= sum + i;
}
```

4. How much time does this code fragment take to execute?

5. How could you replace it with code that runs faster?