CSC 225 Midterm Exam: Summer 1997

1.(a) [10 marks] Define what it means for $f(n) \in O(g(n))$.

(b) [10 marks] Use your definition from (a) to prove that if $S(n) \in O(f(n))$ and $T(n) \in O(g(n))$, and both $S(n)$ and $T(n)$ are strictly positive for all n, then $S(n) * T(n) \in O(f(n) * g(n))$.

2. Consider $S(n) = \sum_{i=1}^{n} i^3$.

   (a) [10 marks] Prove by lower bounding each term of the sum that $S(n) \geq \frac{1}{16} n^4$.

   (b) [10 marks] Prove by upper bounding each term of the sum that $S(n) \leq n^4$.

   (c) [10 marks] Argue that $S(n) \in \Theta(n^4)$.

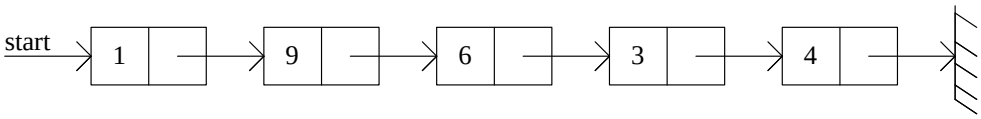
3. For this question, define $T(n) = 1 + 2 * T(n-1)$, $T(0) = 1$.

   (a) [10 marks] Use repeated substitution to express $T(n)$ as a sum. Your sum should not be a recurrence.

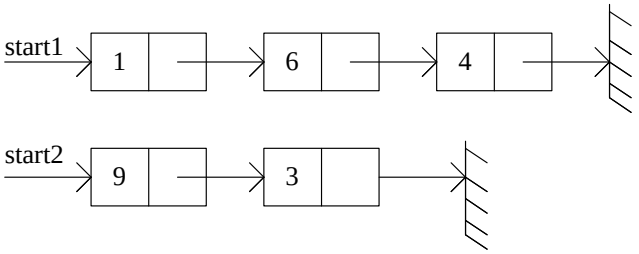
   (b) [10 marks] Give a closed formula (one without a summation sign) for $T(n)$.

   (c) [10 marks] Prove by induction that your answer to part (b) is correct.

4. [30 marks] Give pseudocode for an **iterative** *divide*/*split* function for merge sort which takes as input a linked list L starting at *start*, and returns as output two lists L_1 and L_2 starting at *start*1 and *start*2 respectively. The algorithm should work by placing the first cell from L on L_1, then it should place the second cell from L on L_2, then the third cell from L goes on L_1, and the fourth cell from L goes on L_2, and so on. For example, if the input is:



The output should be:



Be sure to include lots of comments in your pseudocode.