CSC 225 Midterm Exam

Wed. June 27, 1990

1. [15] Fill in the following chart for the comparison model of computation:

Problem	Tight Lower Bound on Number of Comparisons Needed for Worst Case
(a) Search for element in unsorted array	
(b) Search for element in sorted array	
(c) Find max in a sorted array	
(d) Find max in an unsorted array	
(e) Sort n numbers	

For (e), give an answer which is correct up to a multiplicative constant factor. Give exact answers for (a)-(d).

2. [25] Prove that the lower bound you gave for sorting for question #1 is tight. Include a high level description of an appropriate algorithm and justify any results you need regarding the complexity of that algorithm (as we've been doing in class).

3. [25] You work as a programmer for the phone company. You are given a long list of telephone bills and a list of checks from good customers. Consider the following two algorithms for matching customers with bills so we can determine who has not paid.

Approach 1: Do not sort the telephone bills. Do linear search for each of the p bill payments.

Approach 2: Sort the bills and the bill payments by telephone number and do a merge to match customers with bills.

Compare the worst case time complexities. Which approach is the better algorithm?

4. [20] Prove that the solution to the following recurrence relation is $\Theta(k\,2^k)$ by finding appropriate upper and lower bounds.

$$f(k) = k\,2^k + f(k-1), \qquad f(0) = 0$$

5. [20] Define

(a) worst case time complexity of an algorithm,

(b) the comparison model of computation,

(c) internal sort (you do not need to define sort), and

(d) an appropriate basic operation for analysing an algorithm.