**Name: _____**

**ID Number: _____**

# CSC 225 Midterm Exam

## Nov. 2, 1998

**Instructions:**

1. Put your name on every page of the exam.
2. No calculators or other aids. Closed book.
3. Read through the entire exam before beginning. You should have 6 pages including this header page.

| Question | Value | Mark |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 30 | |
| 4 | 30 | |
| **Total** | **100** | |

Recall that you need at least 40% (40/100) in order to write the final exam in this course.

1. [20] Solve the following recurrence using repeated substitution.
   $T(n) = 2n + T(n/2)$, $T(8) = 5$.
   You may assume that $n = 2^k$ for some integer $k \geq 3$.

2. [20] Prove by induction that your solution to question #1 is correct. Or for part marks [10], apply induction to the point where you realize that your solution to #1 is incorrect, and explain what goes wrong.

   The recurrence from Question #1:

   $T(n) = 2n + T(n/2)$, $T(8) = 5$.

   You may assume that $n = 2^k$ for some integer $k \geq 3$.

3.(a) [10] Prove that $\sum_{i=0}^{n} i^6$ is in $O(n^7)$.

(b) [10] Prove that $\sum_{i=0}^{n} i^6$ is in $\Omega(n^7)$.

(c) [10] Prove that $p(n) = 3 + 5\,n + \frac{n^2}{25}$ is not in $O(n)$ .

4. [30] Write detailed pseudocode (almost C code but without worrying about syntax) for Quicksort as described below.

**Input**: List L of values to be sorted.
**Output**: The pointers of L are rearranged so that the values are sorted.

1. **Base case:** If the list has 0 or one elements it is already sorted. Return.
2. **[Divide]**

2.1 Choose x to be the key value in the first element on the list.

2.2. Partition L into 3 lists:

$L_1$- records with key less than x.

$L_2$- records with key equal to x.

$L_3$- records with key greater than x.

3. **[Conquer]** Sort $L_1$, and $L_3$ recursively using Quicksort.
4. **[Merge]** The final list should be $L_1 \; o \; L_2 \; o \; L_3$

To make the merge step more efficient, use
void list_quicksort(start, end)
When called, **start** is set to point to the first element on the list and **end** has no particular value. On return, **start** points to the first element of the sorted list and **end** points to the last one.

This page has been left blank to give you space to finish Question #4.