

CSC 428/528 Midterm Name: \_\_\_\_\_

February 21, 2014 Student Number: \_\_\_\_\_

Total of 40 marks. Closed books and notes; no calculators.

1. [2 marks] What famous computer scientist had their first paper in MAD magazine?

ANSWER: \_\_\_\_\_

2. [3 marks] How many  $n$ -input 2-output boolean functions are there?

ANSWER: \_\_\_\_\_

3. [3 marks] Write  $\langle x, y, z \rangle$  in CNF and in DNF.

ANSWER: CNF: \_\_\_\_\_, DNF: \_\_\_\_\_

4. [3 marks] What is our favorite way of isolating the rightmost 1 bit in a binary number  $x$ ?

ANSWER: \_\_\_\_\_

5. (a) What is the 2-adic representation of  $-1/3$ ?  
(b) What is the 2-adic representation of  $+2/3$ ?

(a) [1 marks] ANSWER: \_\_\_\_\_

(b) [2 marks] ANSWER: \_\_\_\_\_

6. [3 marks] Let  $x = (0000110101111000)_2$ . What is  $\rho x$ ,  $\lambda x$ , and  $\nu x$ ?

ANSWER:  $\rho x =$  \_\_\_\_\_,  $\lambda x =$  \_\_\_\_\_,  $\nu x =$  \_\_\_\_\_.

7. [3 marks] A directed graph is Eulerian if and only if \_\_\_\_\_?

Give a bitstring that represents a De Bruijn cycle for a 3-bit window: \_\_\_\_\_.

8. [3 marks] Convert the boolean chain on the left into a normal boolean chain. Is the function computed by the original chain (i.e., the value of  $x_4$ ) normal?

$$\begin{array}{l|l}
 x_1 & \hat{x}_1 \\
 x_2 & \hat{x}_2 \\
 x_3 = \bar{x}_1 \equiv x_2 & \hat{x}_3 = \\
 x_4 = \bar{x}_3 \wedge x_3 & \hat{x}_4 =
 \end{array}$$

9. [3 marks] Define: An implicant is *prime* if and only if

ANSWER: \_\_\_\_\_

10. [2 marks] What is the value of #3FF0000000000000 if it is interpreted as a IEEE 754 floating point number?

ANSWER: \_\_\_\_\_

11. [3 marks] Express the threshold function  $\llbracket x_1 + 2x_2 + 3x_3 \geq 3 \rrbracket$  as a majority function.

ANSWER: \_\_\_\_\_

12. [3 marks] Fill in the implication below:

**Theorem** (Horn). *The Boolean function  $f(x_1, \dots, x_n)$  is expressible as a conjunction of Horn clauses if and only if*

$f(x_1, \dots, x_n) = f(y_1, \dots, y_n) = 1$  implies \_\_\_\_\_.

for all Boolean values  $x_j$  and  $y_j$ .

13. [6 marks] The following graph arose from applying the book's algorithm to a 2SAT problem (aka Krom clause satisfaction problem).

(a) How do we know that there is a satisfying assignment?

ANSWER: \_\_\_\_\_

(b) What is the big-O time complexity of the underlying algorithm if there are  $n$  variables and  $m$  clauses? Explain briefly.

ANSWER: \_\_\_\_\_

(c) What is a satisfying assignment (give it as a binary string)?

ANSWER:  $abcdef =$  \_\_\_\_\_.

