THE UNIVERSITY OF CALGARY
DEPARTMENT OF COMPUTER SCIENCE

CPSC 453

Mid-Term EXAM

Time Allowed 90 mins. November 8, 2004

Closed Book, calculators are permitted. The questions carry equal weight. Marks will be given for answers to four out of five questions. Show all of your working.

1. (Geometric Transformations)

   (a) Write down the cartesian coordinates, corresponding to the following homogeneous coordinates : $(6,4,2)$ and $(12,8,4)$.

   ### answer

   (x,y,W) *homogeneous*
   To change to Cartesian coordinate in $2D$ divide by W. $(x/W, y/W)$
   Note that there is a dimension reduction. $(6,4,2)$ goes to $(6/2, 4/2) = (3,2)$
   $(12,8,4)$ goes to $(12/4, 8/4) = (3,2)$

   (b) Show that the homogeneous transformation matrix M, below is a special orthogonal matrix:

   $$\begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   ### answer

   See notes on special orthogonal matrices. Three facts that show special orthogonal. In upper $2x2$ matrix.

   i. magnitude of row vectors (and column vectors) are 1
   $\sqrt{\cos(\theta) * cos(\theta) + -sin(\theta) * -sin(\theta)} = 1$
   ii. determinant is 1 $cos(\theta) * cos(\theta) + -sin(\theta) * -sin(\theta) = 1$
   iii. vectors are orthogonal i.e. dot product is zero $(cos(\theta),\ -sin(\theta)).(sin(\theta),\ cos(\theta)) = 1$

   (c) Find the inverse of the above matrix.

   ### answer

   Since the matrix is special orthogonal the inverse is the transpose.

   $$\begin{bmatrix} cos(\theta) & sin(\theta) & 0 \\ -sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   (d) Show that one of the row vectors can be transformed into the Y axis.

   ### answer

   Since the matrix is special orthogonal, one of the properties is that each row vector, written as a column vector, when multiplied by the matrix will produce a unit vector in the direction of the axes.
   e.g. for the 2nd

   $$\begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} sin(\theta) \\ cos(\theta) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

(e) In a viewing system the vector from the eye into the scene is given by $\vec{g}$, and the up vector by 0 1 0. Derive a coordinate system $(\vec{u},\vec{v},\vec{w})$ for viewing and write down the rotation matrix that will align the viewing system with the $(\vec{x},\vec{y},\vec{z})$ world system.

answer

Use the above property see my notes or:

- See page 29 of the text book: Constructing a basis from a single vector.
- See page 97 and 98 concerning arbitrary 3D rotations.
- See page 114 of the text book for the answer to this question.

let the up vector $\vec{t} = (0\ 1\ 0)$

$$\vec{w} = \frac{\vec{g}}{\|\vec{g}\|} \tag{1}$$

$$\vec{u} = \frac{\vec{t} \times \vec{w}}{\|\vec{t} \times \vec{w}\|} \tag{2}$$

$$\vec{v} = \vec{w} \times \vec{u} \tag{3}$$

In the book there is a minus sign in front of the $\vec{w}$ definition, this is there because the book eye coordinate system views along the negative $\vec{w}$ axis.

2. (Ray Tracing)

(a) In a ray tracer intersections may occur beneath the surface instead of exactly on the surface of an object due to numerical imprecision. After a primary ray intersection with triangle A, a shadow ray is sent out towards the light source. If the shadow ray hits the same triangle it is then ignored since it is impossible for the second intersection to be valid. This solution can still produce ray tracer *acne* (black pixels). Describe why this solution is insufficient and suggest a way to overcome the problem.

(Hint: consider adjacent triangles in a mesh).

answer

This algorithm works fine for a single triangle but the problem occurs when the triangle is in a mesh. When the shadow ray is sent out towards the light source, if the next hit is the same triangle it will be ignored (according to the question) and the ray will be properly tested against the light source. If the next hit is with the neighbouring triangle it could also be an error and it would be best to test the distance along the ray of the intersection. If this distance is less than some small ε than the intersection should also be considered to be invalid. (see figure).

(b) A ray, origin $\mathbf{E} = (0\ 0\ 10)$ is fired in the direction of the vector $\mathbf{d} = (0\ -1\ -1)$ at a sphere, origin $(1,\ -6,\ 0)$ and radius 3.0. Using a method that might be implemented in a ray tracer, calculate the point of intersection, of the ray with the sphere. Is this intersection point valid? Show all your working.

answer

See my notes for diagram. Basically substitute the ray equation in the circle equation but some calculations can be saved by following the method in the notes (taken from Graphics Gems I - Academic Press). The answer is a grazing intersection. The discussion is that the value of $disc = 0$ could be taken as a hit or not a hit, either is acceptable. If you want nice images then the sphere image should be anti-aliased by sending more rays around the hit point depending on the sampling method chosen.

- E is the Ray origin (0, 0, 10)
- $\vec{EO}$ is the vector from the ray origin to the sphere origin
- $\vec{d}$ is the ray vector (0 -1 -1)
- $\vec{V}$ is the normalized ray vector (0 -0.707 -0.707)

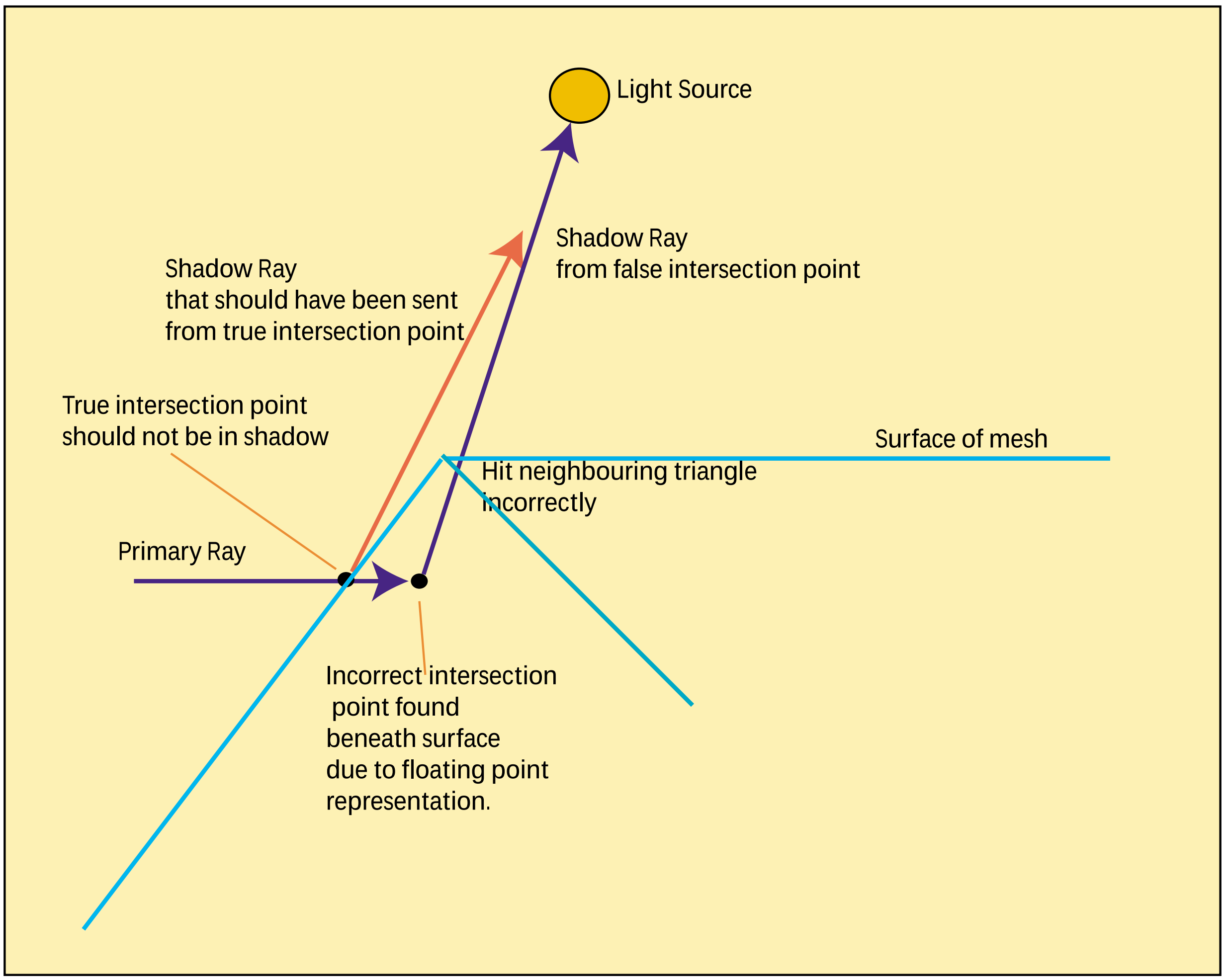

Figure 1: Ray intersection problem due to rounding errors

- p is the intersection point

$$\vec{EO} = (1\ -6\ 0) - (0\ 0\ 10) = (1\ -6\ -10) \tag{4}$$
$$c = \|\vec{EO}\| = 11.70047 \tag{5}$$
$$v = \vec{EO} \cdot \vec{V} = (1\ -6\ -10).(0\ -0.707\ -0.707) \tag{6}$$
$$v = 11.3137 \tag{7}$$
$$v^2 = 128 \tag{8}$$
$$d = \sqrt{(}r^2 - (c^2 - v^2) \tag{9}$$
$$\vec{EO} \cdot \vec{EO} = 137 \tag{10}$$
$$disc = r^2 - (\vec{EO} \cdot \vec{EO}) - v^2 \tag{11}$$
$$disc = 9 - 137 + 128 \tag{12}$$
$$disc = 0 \tag{13}$$
$$p = E + (v - sqrt(disc)) * V) \tag{14}$$
$$p = (0, 0, 10) + (11.3137 - 0) * (0\ -0.707\ -0.707) \tag{15}$$
$$p = (0, -8, 2) \tag{16}$$

3. (Parametric Curves and Generalized Cylinders)

(a) Distinguish between parameter space and modeling (object) space.

answer

Basis functions are defined in parametric space. For a particular value of t a point on the object space curve is defined by weighting each control point (in object space) by the appropriate basis function. The curve is drawn by finding a series of points along the curve as t varies along the unit interval (for uniform parametric curves).

(b) Given the four basis functions ($b_i(t)$ $0 \leq i \leq 3$) and ($0 \leq t \leq 1$) and four points, (P_i $0 \leq i \leq 3$). Find an expression for the desired curve in the modelling space.

answer

$\sum P_i * b_i(t)$

(c) A generalized cylinder is built around a cubic curve given by the parametric equations:

$f_x(t) = 4t^3 - 6t^2 + 4t + 1$

$f_y(t) = 6t^3 - 9t^2 + 2t + 2$

Derive expressions for the Frenet frame at the point where $t = 0$. (In other words don't evaluate the expressions).

answer

find the tangent vector and acceleration vector at the point 0.0 I have also added the values at 0,5 into the table for the next part of the question.

$function$	$t = 0$	$t = 0.5$
$f_x(t) = 4t^3 - 6t^2 + 4t + 1$	1	2
$f_y(t) = 6t^3 - 9t^2 + 2t + 2$	2	0
$1stderivative$	$t = 0$	$t = 0.5$
$f'_x(t) = 12t^2 - 12t + 4$	4	-2
$f'_y(t) = 18t^2 - 18t + 2$	2	-2.5
$12ndderivative$	$t = 0$	$t = 0.5$
$f''_x(t) = 24t - 12$	-12	0
$f''_y(t) = 36t - 36$	-36	0

At $t = 0$ we have:

$$\vec{v} = (4,2) \tag{17}$$
$$\vec{q} = (-12,-36) \tag{18}$$
$$\vec{T} = \frac{\vec{v}}{\|\vec{v}\|} \tag{19}$$
$$\vec{B} = \frac{\vec{v}\times\vec{q}}{\|\vec{v}\times\vec{q}\|} \tag{20}$$
$$\vec{N} = \frac{\vec{v}\times\vec{q}\times\vec{v}}{\|\vec{v}\times\vec{q}\times\vec{v}\|} \tag{21}$$

(d) What problem is encountered if the frame is calculated at $t = 0.5$? Describe briefly how the problem can be overcome in an algorithm for building the cylinder from circular cross sections.

answer

It can be seen from the above that the vector formed from the 2nd derivative is zero. This means that there is a pointat which the Frenet frame cannot be calculated analytically as indicated above.

A rotation minimising frame is used (see notes). After finding an initial frame successive frames are found by rotating the previous frame around an axis. If the frame at point p_i is known then the frame at point p_{i+1} is found by rotating around an axis. The axis of rotation is derived from the following cross product:

$$\vec{T_i} \times \vec{T_{i+1}}$$

If this is zero the frame is simply translated to p_{i+1}. The angle of rotation is found from the dot product:

$$\vec{T_i} \cdot \vec{T_{i+1}} = \|\vec{T_i}\|\|\vec{T_{i+1}}\|cos(\alpha)$$

$\vec{B_{i+1}}$ and $\vec{N_{i+1}}$ are computed by rotating $\vec{B_i}$ and $\vec{N_i}$

4. (Shading)

(a) In a rendering system using Phong Shading, attenuation of intensity with distance to a light source has been implemented, why is it not accurate to use an inverse square function (e.g. 1/distance*distance) in the illumination calculation?

answer

Light energy falls off with the square of the distance. See first shading slide.

(b) Joe Hackquick has built a very good polyhedral approximation to a sphere consisting of 100 Gouraud shaded triangles. Joe's renderer uses the Phong light model and Joe wants to obtain a shiny image. The sphere is placed at the origin and the viewer has been placed at the point $5,5,5$. Where should the light source be placed to view the surface. Even with the viewer and light source placed optimally the surface does not have a bright specular, why not?

answer

Assuming that the viewer is looking along the normal to the sphere then the light source should be positioned at the same point. Joe will not see a bright specular because Gouraud shading averages the intensity of the mesh and only calculates the intensity from the Phong light model at the triangle vertices.

(c) A surface at the origin has a normal vector which points up the $y-axis$. A light source is placed at the point $(-a,\ b,\ c)$. Calculate the vector in the direction where the highlight will be maximum.

answer

See the diagram in the notes (shading page 7). $\vec{L}$ is the vector to the light source and $\vec{V}$ is the vector to the viewer. $\vec{N}$ is the normal and $\vec{R}$ a vector in the reflection direction. It is shown in the notes that:

$$\vec{R} = 2(\vec{N} \cdot \vec{L})\vec{N} - \vec{L}\vec{R} = 2((0\ 1\ 0) \cdot (-a\ b\ c))(0\ 1\ 0) - (-a\ b\ c)$$

5. (General CG Questions)

   (a) What is twice the answer to Life the Universe and everything?

   (b) (Graph Structures) Design a simple 2D model of a car. The car consists of a body and 4 wheels. The wheel is consists of 8 spokes (lines) and a circle. The car body consists of 4 seats and an engine. Draw a diagram and describe the data structure and transformations you use. Use rectangles, circles or lines to represent the car parts. You can assume the traversal algorithm given in class will be used on the data structure.

   answer

   (c) (Ray Tracing) Compare and contrast jittering and recursive pixel subdivision as methods of anti-alising in ray tracing.

   answer

   First, see diagams in notes, reproduce these figures. Recursive pixel subdivision: Fires rays as indicated in the figure rays tend to cluster around edges. More samples per pixel where the samples are required.

   Jittering: Displace uniformly distributed rays to sample non-uniformly over a pixel. The jitterring introduces some noise into the image. In general better results are achieved for fewer rays sent per pixel. Noise in the image is more acceptable than aliasing artifacts.

   Jittering

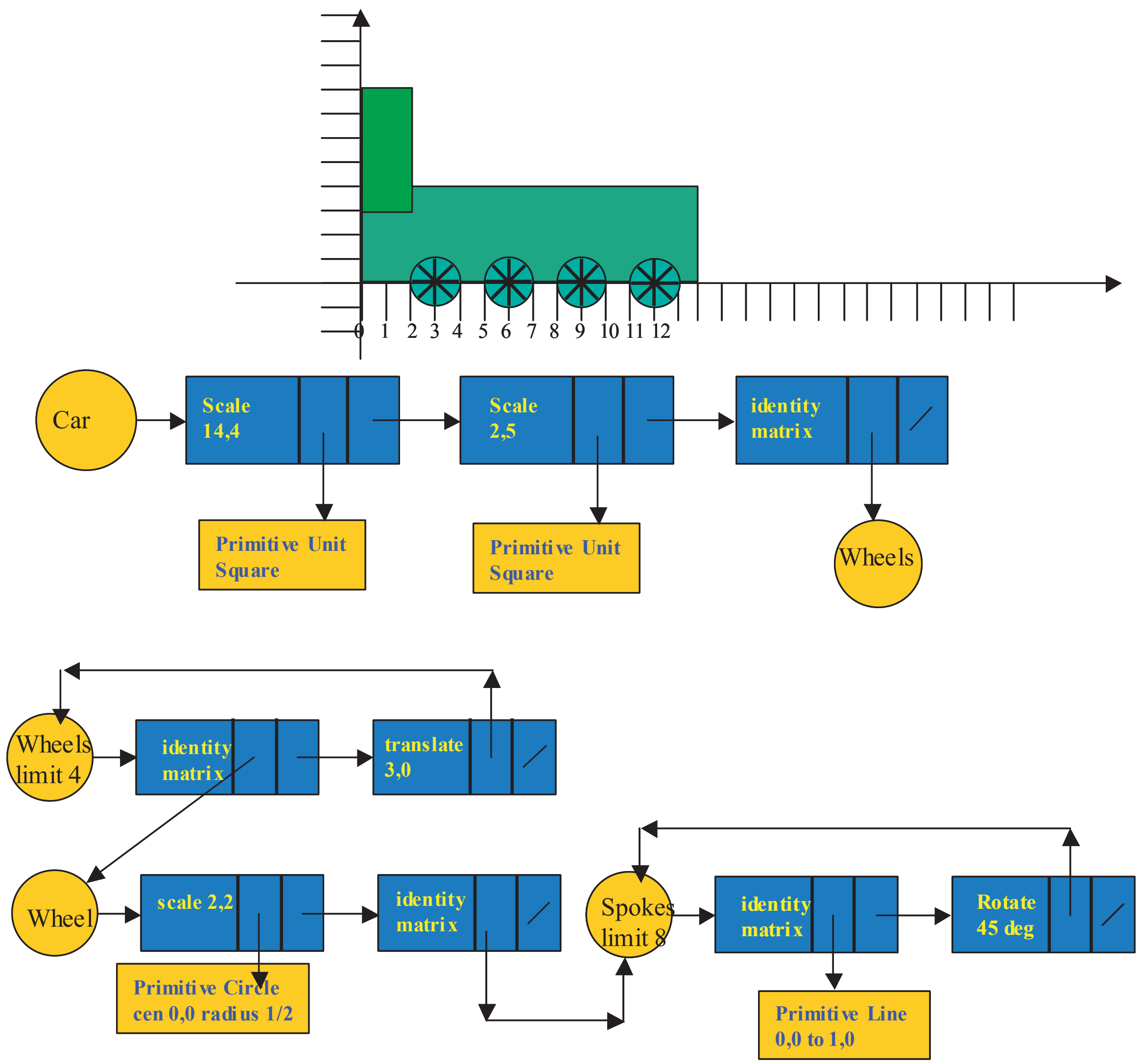

Figure 2: Car Data Structure