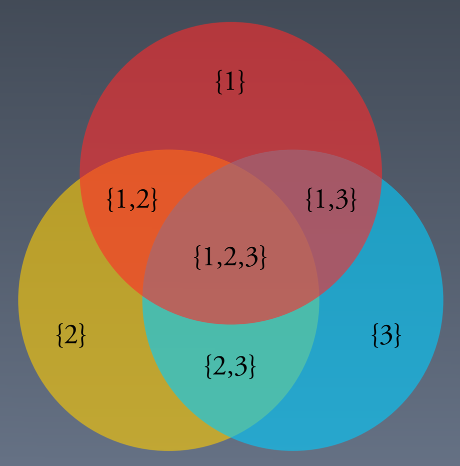
A New Rose : The First Simple Symmetric 11-Venn Diagram

Venn Diagrams : A ***n*-Venn** diagram is a collection of n closed curves in the plane (intersecting at finitely many points) with 2^n distinct regions, where each region is in the interior of a unique subset of the curves.



A n-Venn diagram is ***symmetric*** if a rotation of the plane by $2\pi/n$ radians leaves the diagram fixed (up to a relabeling of the curves).

A ***k*-region** is a region that is in the interior of exactly k curves.

Monotone Venn diagram : Every k-region is adjacent to at least one (k-1)-region (if $k > 0$) and it is also adjacent to at least one (k+1)-region (if $k < n$).

Polar-symmetric : The same diagram is obtained if you turn the diagram inside-out.

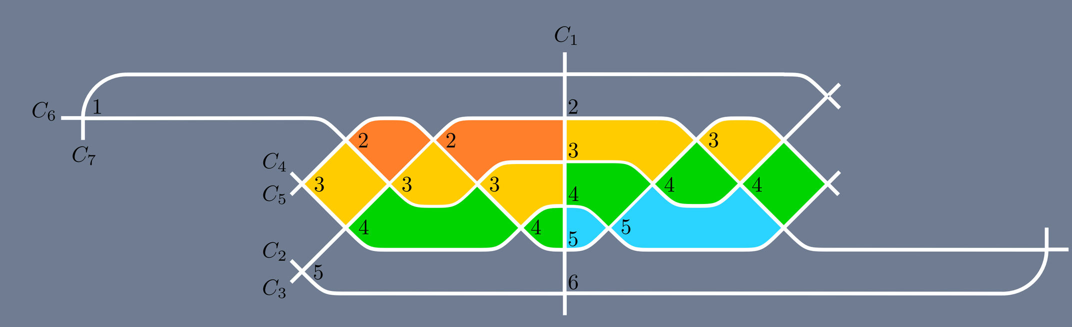
Representation : Let π be the vector of curves labels along a ray emanating from the center as we sweep the diagram in clockwise order. The ***crossing sequence*** is a sequnce of length $(2^n - 2)/n$, where an entry of value i shows a crossing of curves $\pi[i]$ and $\pi[i+1]$.

Theorem

A simple monotone symmetric n-Venn diagram is crosscut-symmetric iff it can be represented by the crossing sequence $\rho, \alpha, \delta, \alpha^{r+}$ where

- $\rho = 1, 3, 2, 5, 4, \ldots, n-2, n-3$ *and* $\delta = n-1, n-2, \ldots, 3, 2.$
- $|\alpha| = |\alpha^{r+}| = (2^{n-1} - (n-1)^2)/2$ *and* $\alpha[i] \in \{2, \ldots n-3\}.$
- α^{r+} *is obtained by reversing* α *and incrementing each element by 1.*

Crossing sequence of 7-Venn M4 : [1,3,2,5,4,3,2,3,4,6,5,4,3,2,5,4,3,4]



A cluster of crosscut-symmetric 7-Venn M4.

C — **A *cluster*** is a connected component of $(2^n-2)/n$ regions of the diagram (excluding the outermost and the innermost regions) that can be used to produce the diagram by applying successive rotations of $2\pi/n$ radians about a center point in the plane. A cluster of 7-Venn M4 is shown by the highlighted curve segments.

E — **Crosscut Symmetry :** A symmetric n-Venn diagram is ***crosscut-symmetric*** if it contains a cluster with a crosscut such that for any curve C not containing the crosscut, the sequence of curves intersecting C in the cluster is palindromic.

C_5 Sequence of 7-Venn M4 : [C_4,C_6,C_3,C_6,C_4,C_1,C_4,C_6,C_3,C_6,C_4]

B — Symmetric n-Venn diagrams exist only when n is prime [**Henderson** 1963]. The first non-simple symmetric 11-Venn diagram was discovered by **Hamburger** in 1999. In 2002 **Griggs**, **Killian** and **Savage** showed that symmetric Venn diagrams exist for any prime number of curves. Their diagrams however are highly non-simple. For n=2,3 and 5, there is only one simple symmetric n-Venn diagram. There are 23 simple symmetric 7-Venn Diagrams that are monotone.

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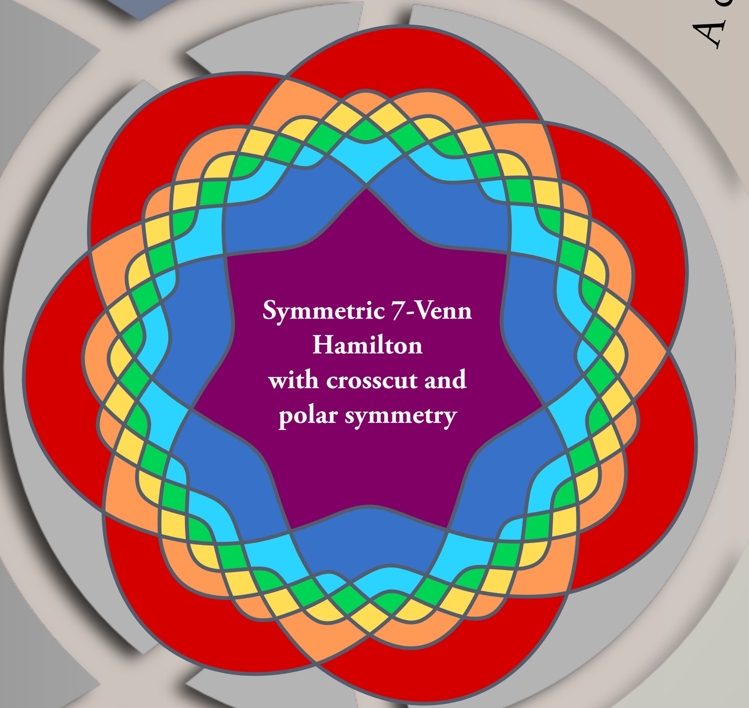

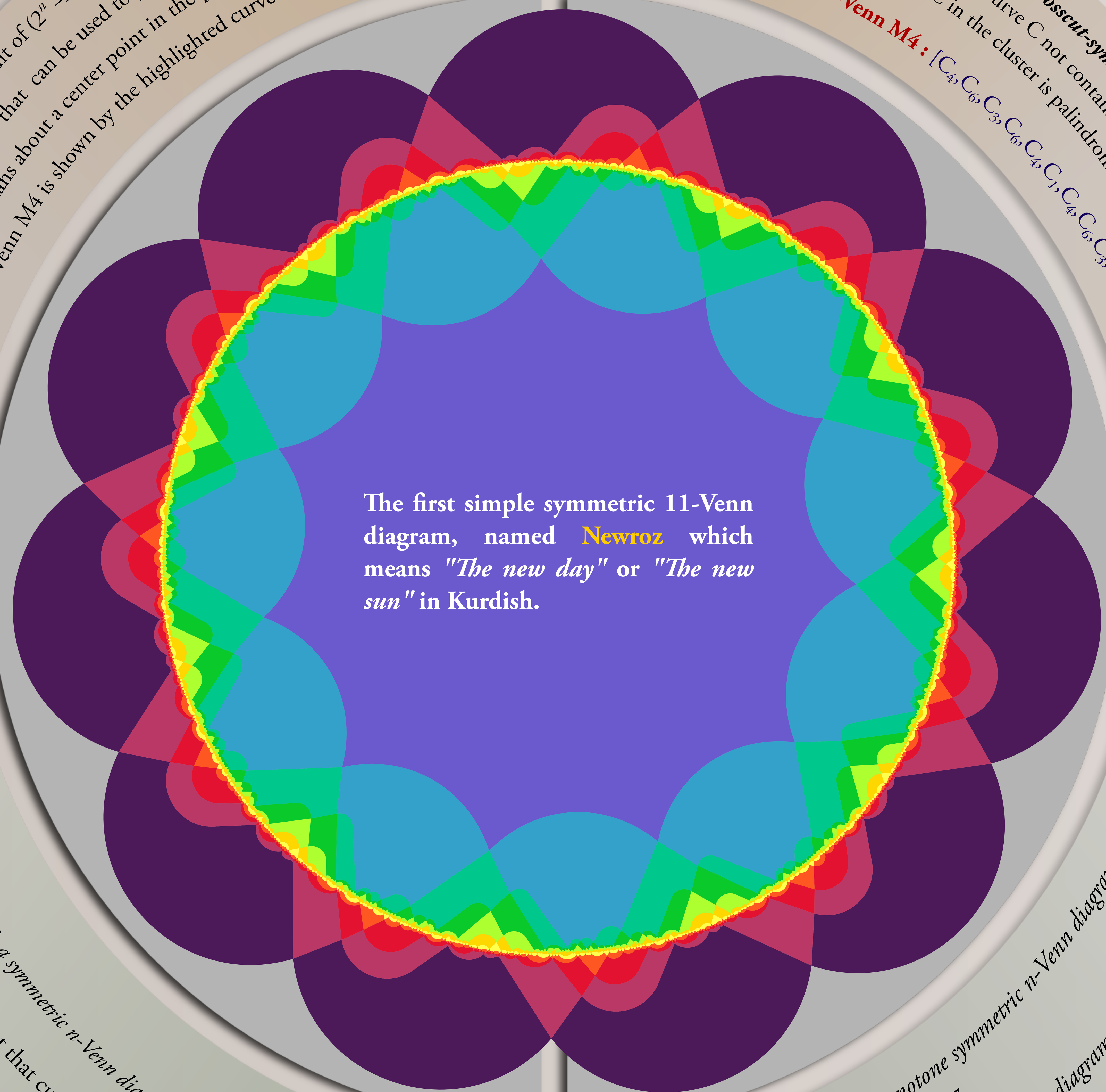

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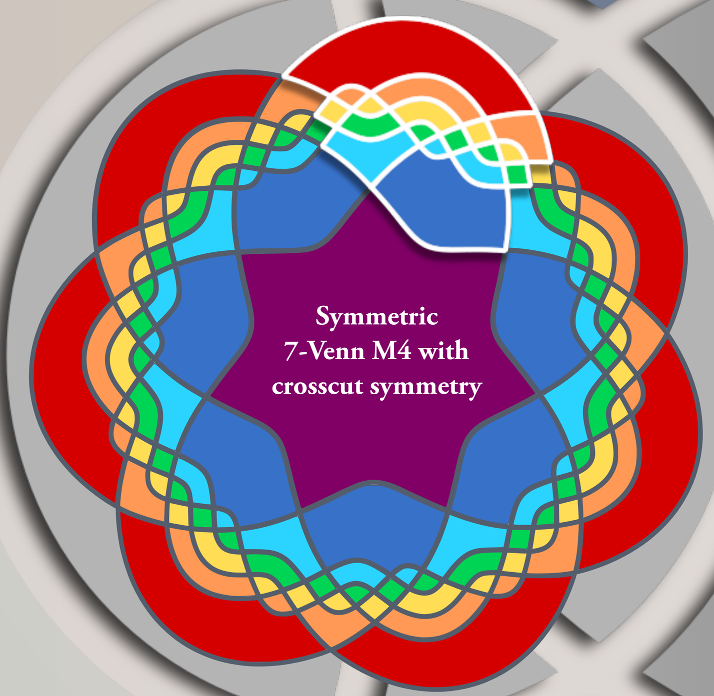


H — The **5-elipses** Venn diagram, discovered by **Branko Grünbaum**, is the only simple symmetric 5-Venn diagram. It is also polar-symmetric and crosscut-symmetric. The background image of this poster is a cluster of 7-Venn **Hamilton** discovered by **Anthony Edwards** in 1992. The only other simple symmetric monotone 7-Venn diagram with crosscut symmetry is **M4**. It was discovered by **Frank Ruskey** in 1996.

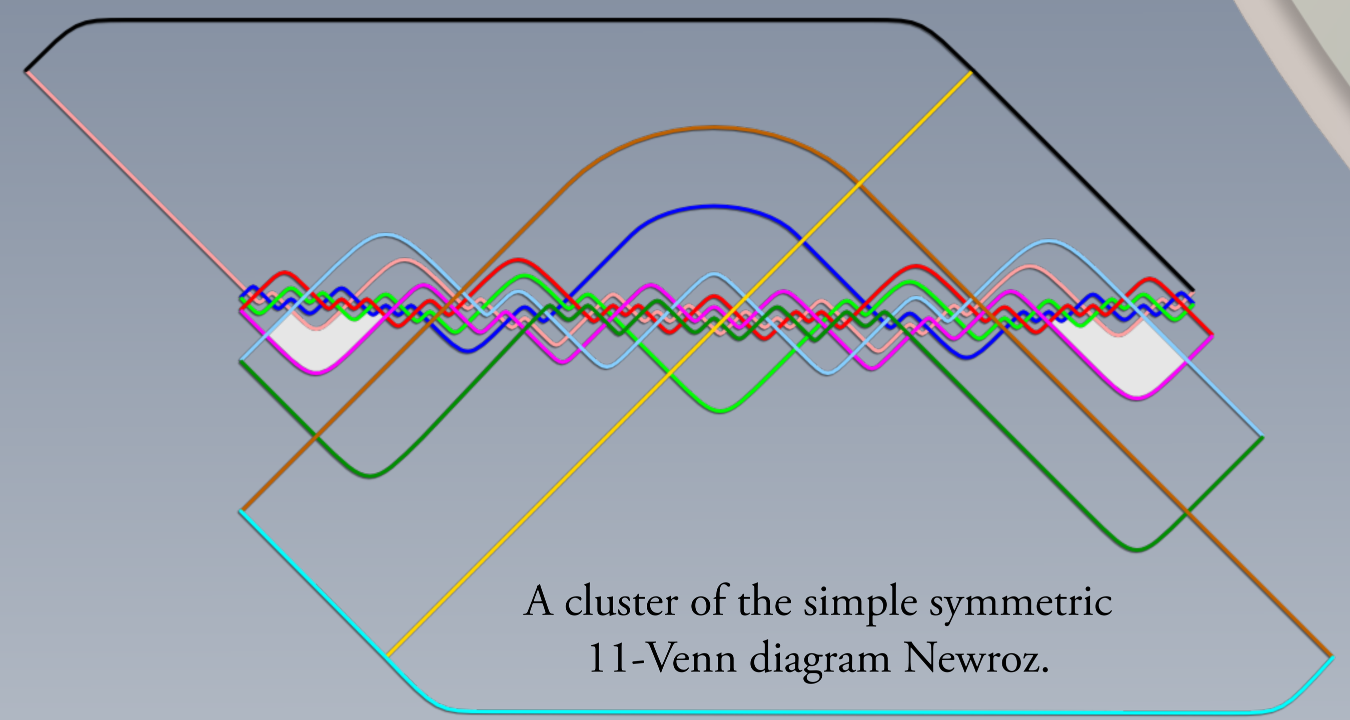
G — **Crossing sequence of Grünbaum's 5-elipses :**

Searching Algorithm

For each possible sequence α

- Construct the crossing sequence $\rho, \alpha, \delta, \alpha^{r+}$.
- Cut-off the search as soon as a duplicate region is found.
- Check if the sequence satisfies Venn diagram and symmetry constraints.

I — For every region on the left side of the crosscut there is a region on the right side such that the sets of containing curves of both regions differ only in the curve that the crosscut belongs to.



A cluster of the simple symmetric 11-Venn diagram Newroz.

α = [323434543234345434545654565676543254346545676787656543457654658765457656876546576567]

D — **Lemma :** *For n > 3, a symmetric n-Venn diagram has at most one crosscut per curve.*

A ***crosscut*** is a curve segment that cuts every other curve sequentially without repetition.

J — **Theorem :** *There is no simple monotone symmetric n-Venn diagram with crosscut and polar symmetry for n>7.*

Open problem: *Is there a simple symmetric n-Venn diagram with polar symmetry for n>7?*

This research is a joint work of Khalegh Mamakani and Frank Ruskey.

For more information see : http://webhome.cs.uvic.ca/ ~ruskey/Publications/Venn11/Venn11.html

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